Computationally efficient stress reconstruction for nonlinear VFM

Bojan Starman, Andraž Maček, Miroslav Halilovič,
Sam Coppieters

University of Ljubljana, Faculty of Mechanical Engineering, Slovenia
KU Leuven, Department of Materials Engineering, Belgium

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Who we are…

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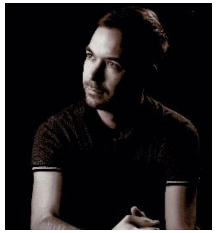
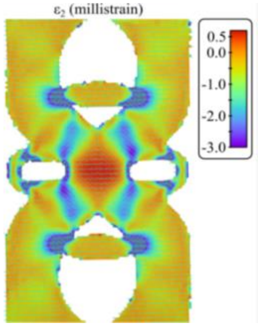


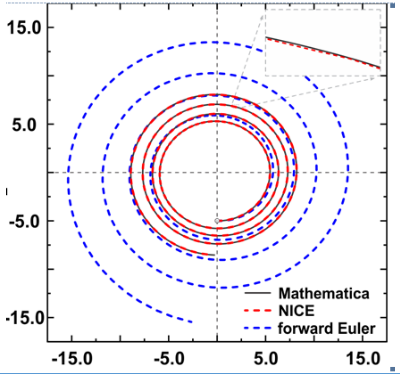
VForm-xSteels

Material modelling & characterization

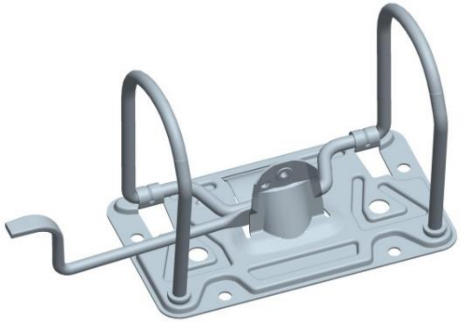


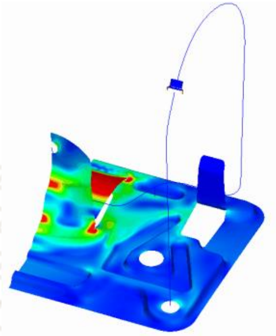
Numerical methods & implementations of material models

- metals
- polymers
- composites



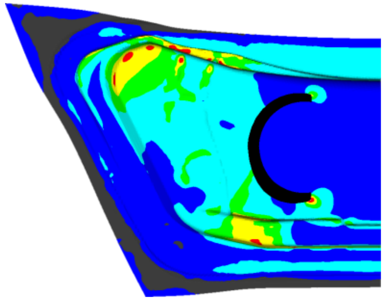
Product development & optimization



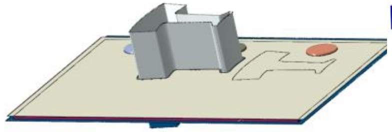


Simulations of technological processes

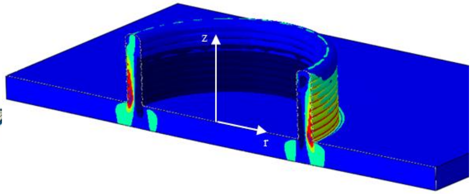
deep drawing, stamping



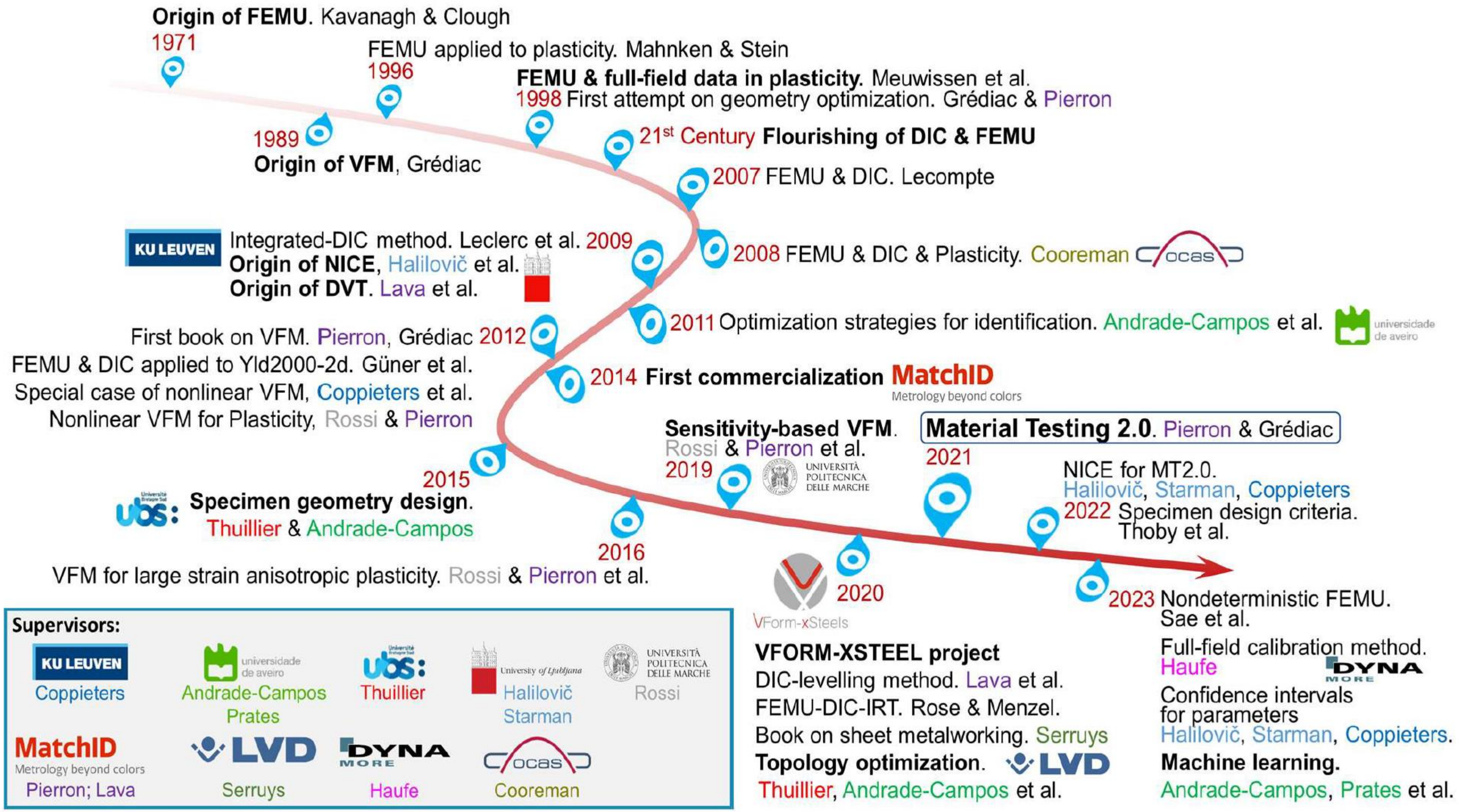
blanking



3D printing

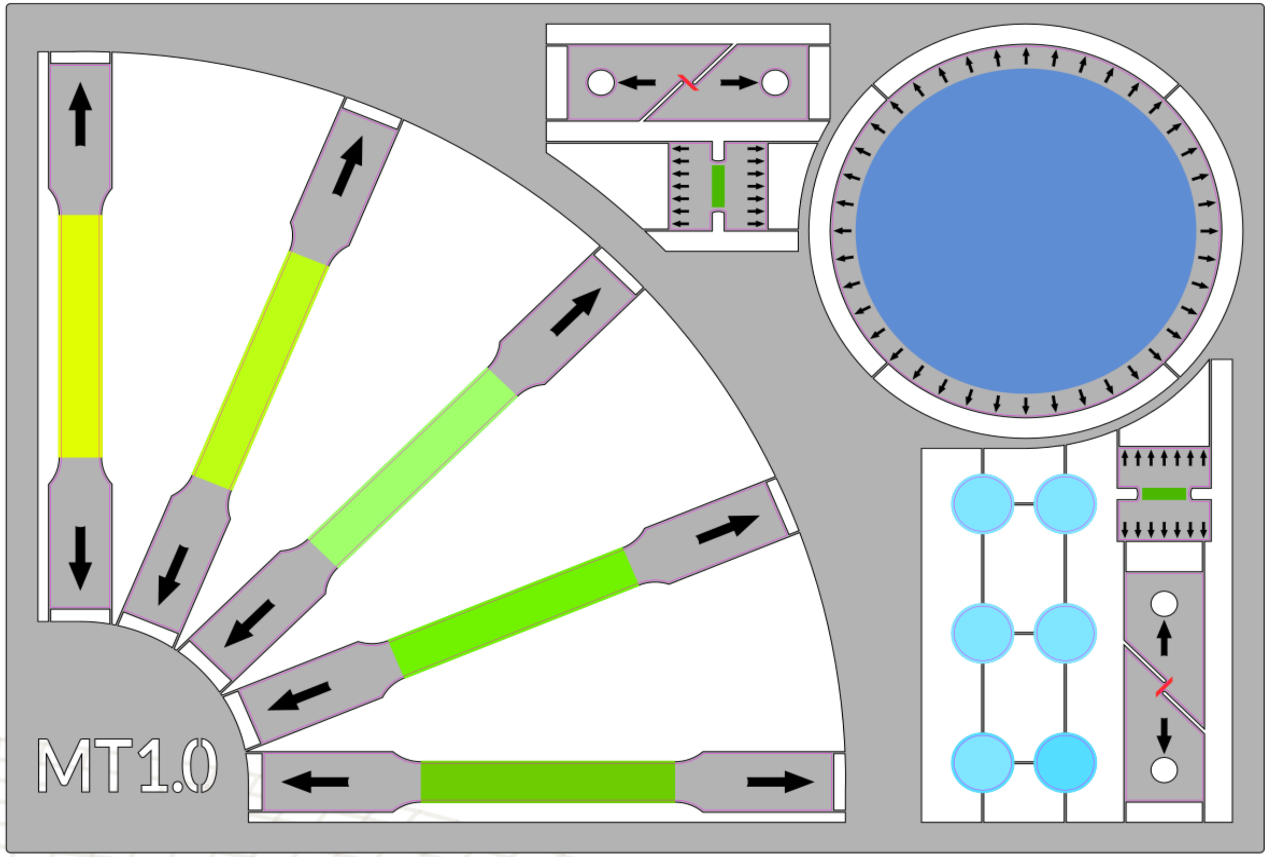


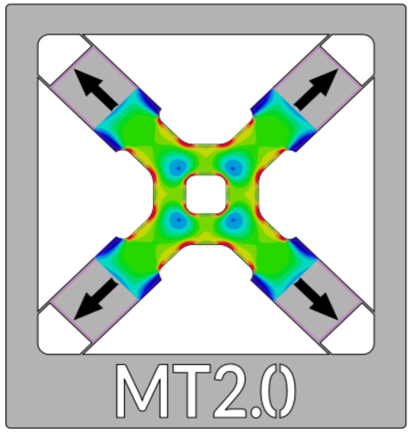
A history of MT 2.0…



MT 1.0 vs MT 2.0 in a nutshell

Material testing 1.0 (MT 1.0) vs Material testing 2.0 (MT 2.0):





Making the material characterization efficient, reliable and robust!

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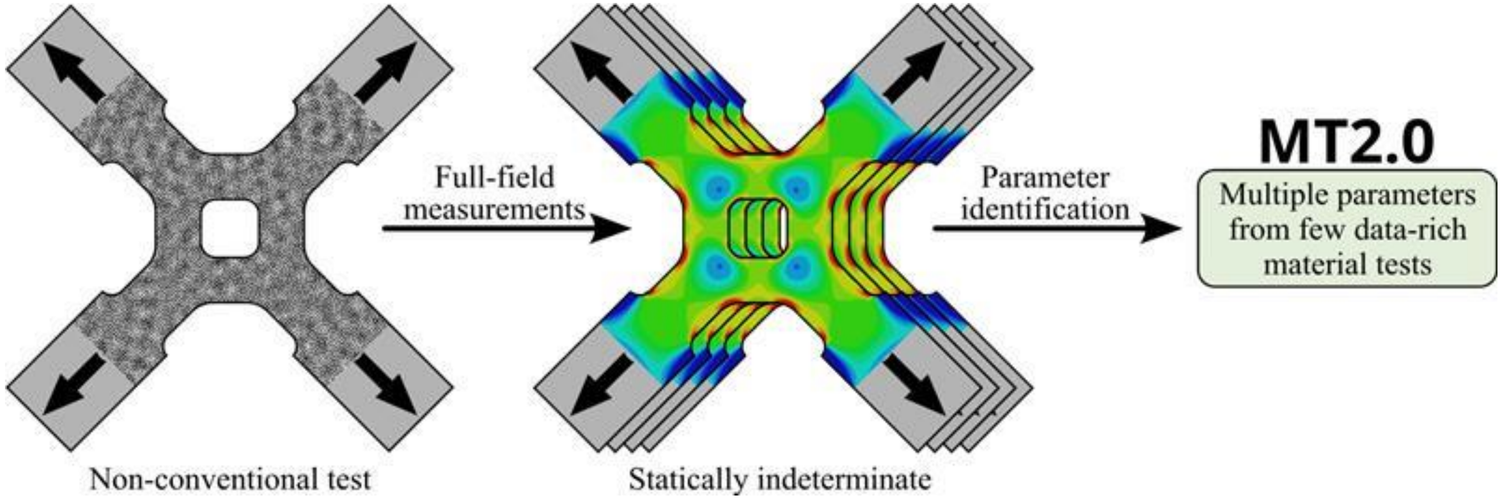

CONSTITUTIVE MODEL

DIC → u_i → F_{ij} → ε_{ij} —model parameters→ σ_{ij}

kinematic consistency

equilibrium

inversely identified by VFM

BOTTLENECK → time consuming stress reconstruction

Stress reconstruction algorithm: a bottleneck of parameter identification

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- Backward Euler (BE)
  - implicit integration scheme
  - consistency condition fulfilled
  - stable
  - solution of system of nonlinear algebraic equations is required
  - iteration procedure in each increment is needed
  - difficult implementation of complex constitutive models
- Next Increment Corrects Error (NICE)
  - explicit integration scheme
  - no drifting from the consistency condition
  - stable
  - fast, efficient
  - simple implementation

NICE: an explicit numerical scheme for integration of constitutive equations, which efficiently combine <u>the implementation simplicity</u> of the forward-Euler scheme and <u>accuracy of the backward-Euler scheme</u>

A class of elastic-plastic constitutive models (system of DAE):

algebraic constraint/equation:

$$\Phi = \Phi\left(\sigma_{ij}, \sigma_{\mathrm{Y}}, \kappa_1, \ldots, \kappa_p\right) = 0$$ (consistency condition)

differential (evolution) equations:

$$\mathrm{d}\sigma_{ij} = C_{ijkl}\,\mathrm{d}\varepsilon_{kl}^{\mathrm{e}} = C_{ijkl}\left(\mathrm{d}\varepsilon_{kl} - \mathrm{d}\varepsilon_{kl}^{\mathrm{p}}\right)$$ (Hook's law)

$$\mathrm{d}\varepsilon_{ij}^{\mathrm{p}} = \frac{\partial\psi}{\partial\sigma_{ij}}\,\mathrm{d}\lambda$$ (flow rule)

$$\sigma_{\mathrm{Y}}\mathrm{d}\varepsilon_{\mathrm{eq}}^{\mathrm{p}} = \sigma_{ij}\,\mathrm{d}\varepsilon_{ij}^{\mathrm{p}}$$ (uniaxial-multiaxial plastic work equivalence)

$$\mathrm{d}\kappa_r = \mathrm{d}\kappa_r\left(\sigma_{ij}, \sigma_{\mathrm{Y}}, \kappa_1, \ldots, \kappa_p, \mathrm{d}\varepsilon_{ij}^{\mathrm{p}}\right); \quad r \in \{1, 2, \ldots, p\}$$ (additional state variables are supposed to follow specified differential laws)

+ loading/unloading conditions

NICE (Next Increment Corrects Error) scheme:

How can we fulfill the algebraic constraint with explicit scheme?

algebraic constraint/equation:

$\Phi = \Phi(\mathbf{\Sigma}) = 0$ → **Taylor power series expansion of the consistency condition on all unknown state variables** → $\Phi_n + \Delta\Phi = 0$

differential (evolution) equations:

$\Delta\mathbf{\Sigma} = \mathbf{C} \cdot \Delta\boldsymbol{\varepsilon} - \mathbf{R}\,\Delta\lambda$ → treated according to the forward-Euler approach → $\Delta\mathbf{\Sigma} = \mathbf{C} \cdot \Delta\boldsymbol{\varepsilon} - \mathbf{R}_n\,\Delta\lambda$

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NICE (Next Increment Corrects Error) scheme:

NICE scheme:

$$\Phi_n + \Delta\Phi = \Phi_n + \left.\frac{\partial\Phi}{\partial\boldsymbol{\Sigma}}\right|_n \cdot \Delta\boldsymbol{\Sigma} = 0$$

$$\Delta\boldsymbol{\Sigma} = \boldsymbol{C} \cdot \Delta\boldsymbol{\varepsilon} - \boldsymbol{R}_n \, \Delta\lambda$$

explicit expression of plastic multiplier

$$\Delta\lambda = \frac{\Phi_n + \left.\frac{\partial\Phi}{\partial\boldsymbol{\Sigma}}\right|_n \cdot \boldsymbol{C} \cdot \Delta\boldsymbol{\varepsilon}}{\left.\frac{\partial\Phi}{\partial\boldsymbol{\Sigma}}\right|_n \cdot \boldsymbol{R}_n}$$

Forward-Euler:

$$\Delta\Phi = \left.\frac{\partial\Phi}{\partial\boldsymbol{\Sigma}}\right|_n \cdot \Delta\boldsymbol{\Sigma} = 0$$

$$\Delta\boldsymbol{\Sigma} = \boldsymbol{C} \cdot \Delta\boldsymbol{\varepsilon} - \boldsymbol{R}_n \, \Delta\lambda$$

explicit expression of plastic multiplier

$$\Delta\lambda = \frac{\left.\frac{\partial\Phi}{\partial\boldsymbol{\Sigma}}\right|_n \cdot \boldsymbol{C} \cdot \Delta\boldsymbol{\varepsilon}}{\left.\frac{\partial\Phi}{\partial\boldsymbol{\Sigma}}\right|_n \cdot \boldsymbol{R}_n}$$

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Simple mathematical example: 2-D loading of membrane

algebraic constraint/equation:

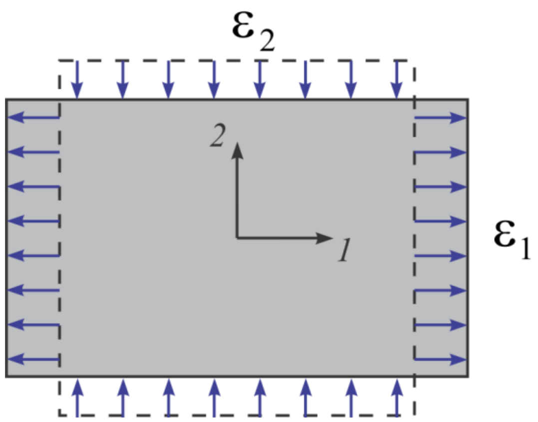
$$\Phi(\sigma_1,\sigma_2,\sigma_Y)=\frac{\sigma_1^2+\sigma_2^2}{\sigma_Y^2}-1=0$$

differential (evolution) equations:

$$\mathrm{d}\sigma_1=E\,\mathrm{d}\varepsilon_1-E\frac{\partial\Phi}{\partial\sigma_1}\mathrm{d}\lambda$$

$$\mathrm{d}\sigma_2=E\,\mathrm{d}\varepsilon_2-E\frac{\partial\Phi}{\partial\sigma_2}\mathrm{d}\lambda$$

$$\mathrm{d}\sigma_Y=H\,\mathrm{d}\lambda$$



Simple mathematical example: 2-D loading of membrane

algebraic constraint/equation:

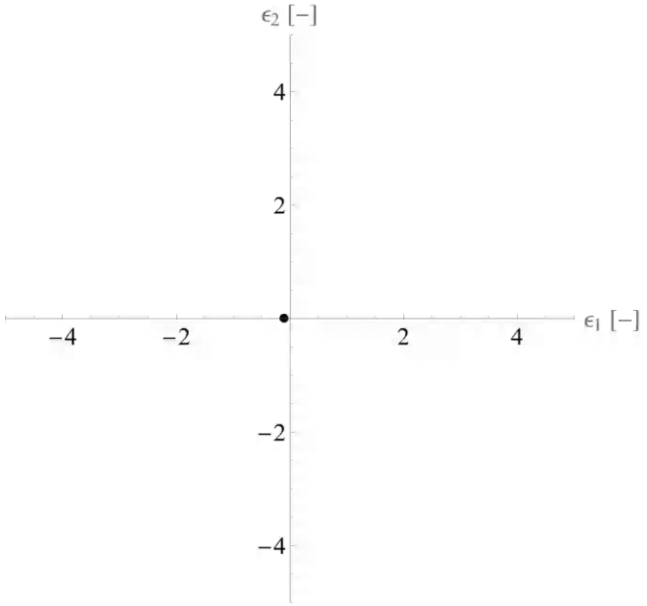
$$\Phi_n + \Delta\Phi = \Phi_n + \left.\frac{\partial\Phi}{\partial\boldsymbol{\Sigma}}\right|_n \cdot \Delta\boldsymbol{\Sigma} = 0$$

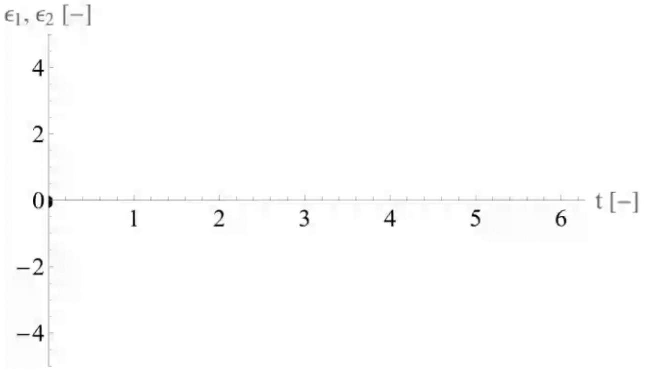
differential (evolution) equations:

$$\Delta\boldsymbol{\Sigma} = \boldsymbol{C}\cdot\Delta\boldsymbol{\varepsilon} - \boldsymbol{R}_n\,\Delta\lambda$$

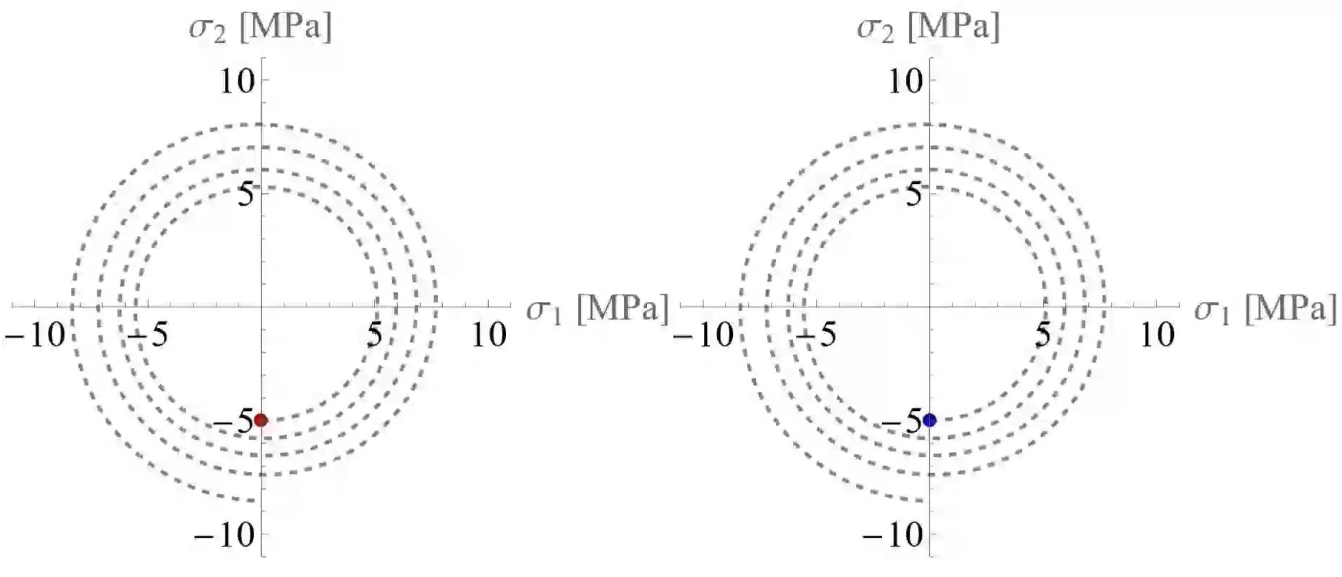
$$\Delta\lambda = \frac{\Phi_n + \left.\frac{\partial\Phi}{\partial\boldsymbol{\Sigma}}\right|_n \cdot \boldsymbol{C}\cdot\Delta\boldsymbol{\varepsilon}}{\left.\frac{\partial\Phi}{\partial\boldsymbol{\Sigma}}\right|_n \cdot \boldsymbol{R}_n}$$

$$\Delta\boldsymbol{\varepsilon} = \begin{Bmatrix}\Delta\varepsilon_1\\ \Delta\varepsilon_2\end{Bmatrix};\ \Delta\boldsymbol{\Sigma} = \begin{Bmatrix}\Delta\sigma_1\\ \Delta\sigma_2\\ \Delta\sigma_Y\end{Bmatrix};\ \boldsymbol{C} = \begin{bmatrix}E & 0\\ 0 & E\\ 0 & 0\end{bmatrix};\ \boldsymbol{R} = \begin{Bmatrix}2E\sigma_1\sigma_Y^{-2}\\ 2E\sigma_2\sigma_Y^{-2}\\ -H\end{Bmatrix}$$





Simple mathematical example: 2-D loading of membrane



Next Increment Corrects Error (NICE)

- explicit integration scheme
- no drifting from the consistency condition
- stable

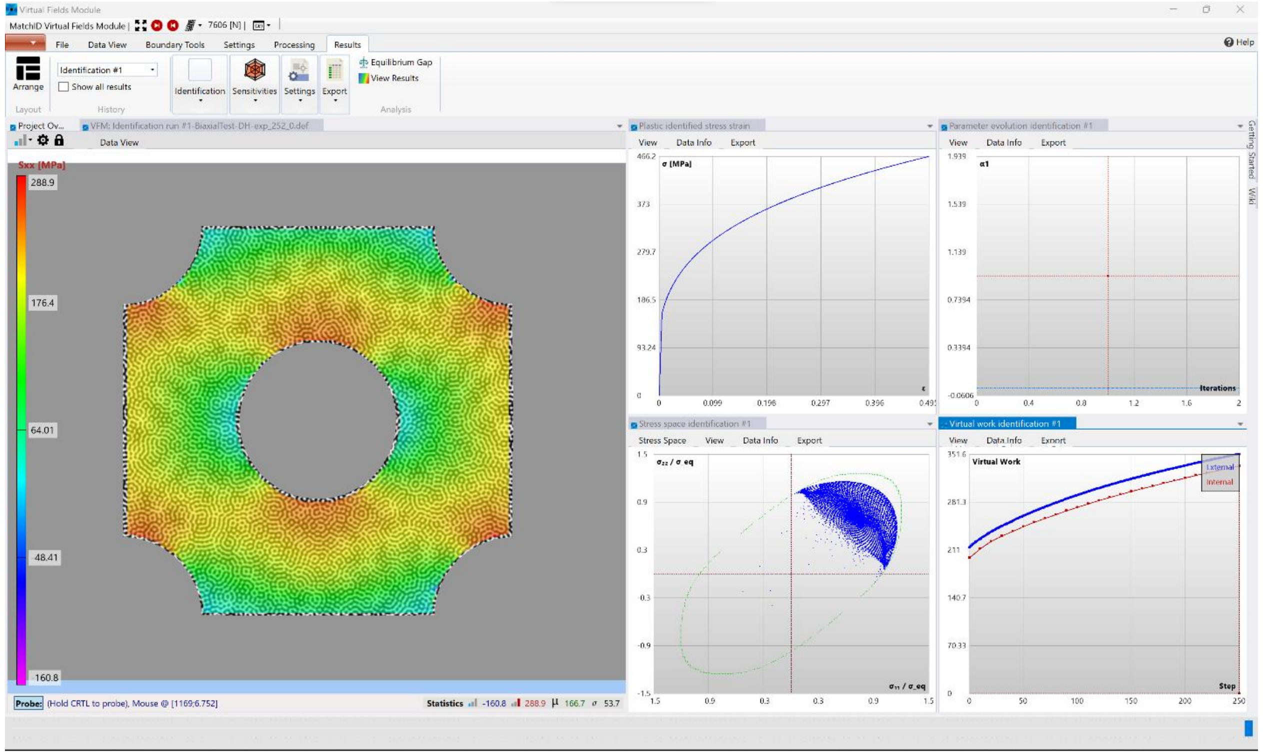
- fast, efficient
- simple implementation
- ✘ substepping for large increments since the error can get large (implicit FEM)

Application to DIC and VFM

Boosting the efficiency of non-linear VFM by implementation of NICE

NICE

MatchID
Metrology beyond colors

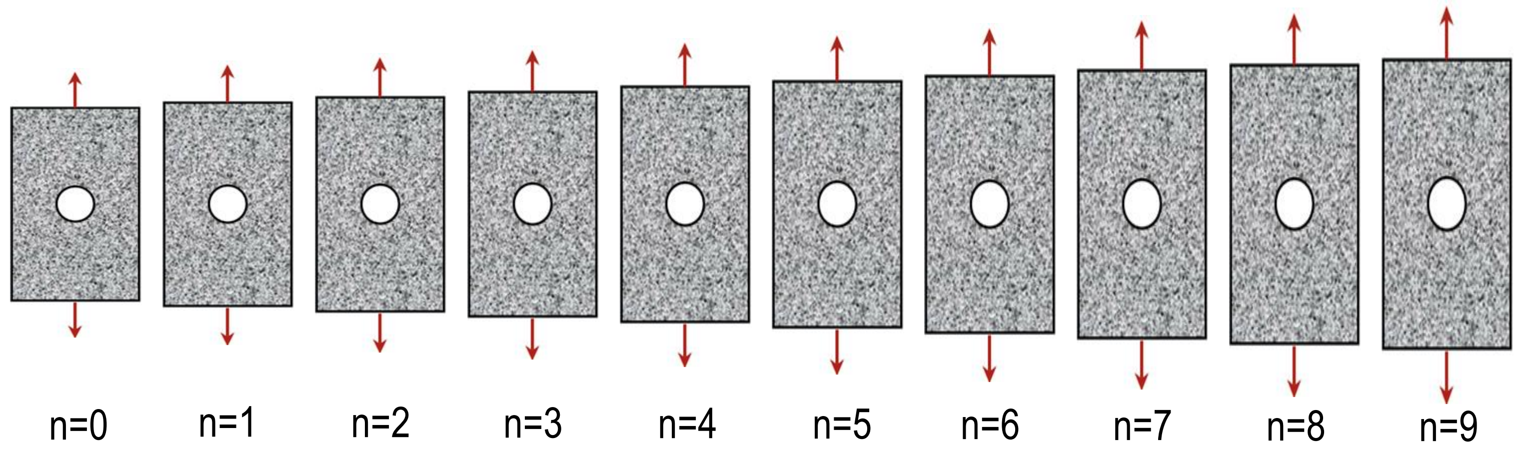


user interface: measure and postprocess full-field test data, apply boundary cond., material model, choose parameters, optimization algorithm, initial conditions, few details and run!

Effect of temporal down-sampling

Image acquisition for DIC

Linear elasticity



No. of frames

The user has control on frame acquisition rate → **improved efficiency?**

Effect of temporal down-sampling

Image acquisition for DIC

Linear elasticity

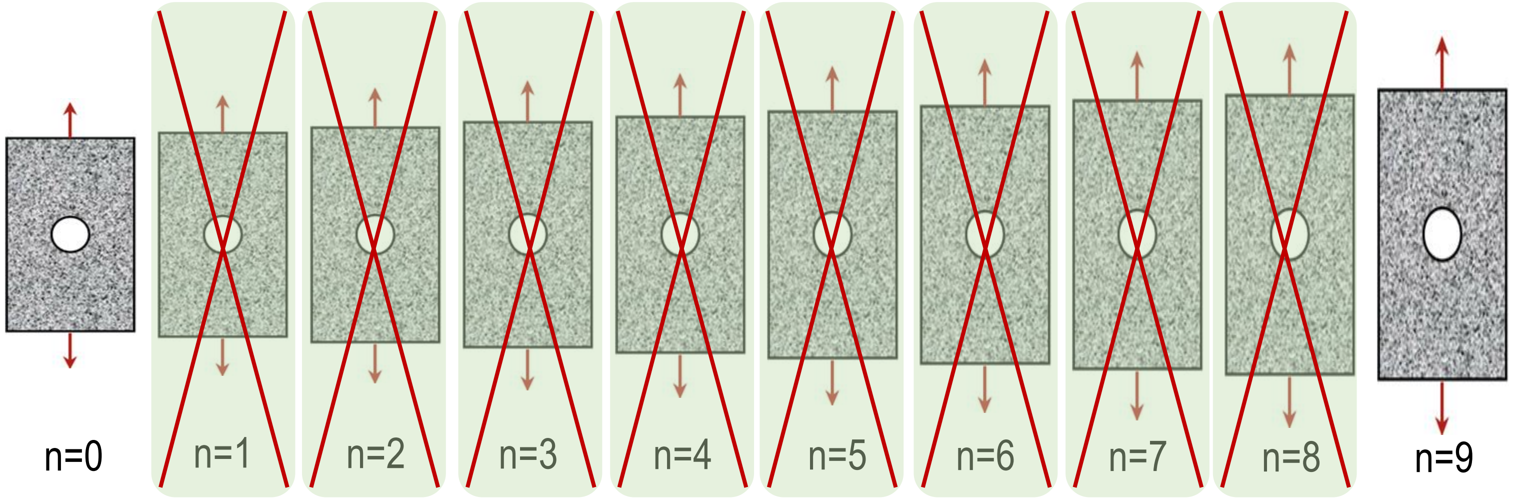


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Image acquisition for DIC

Linear elasticity ✓

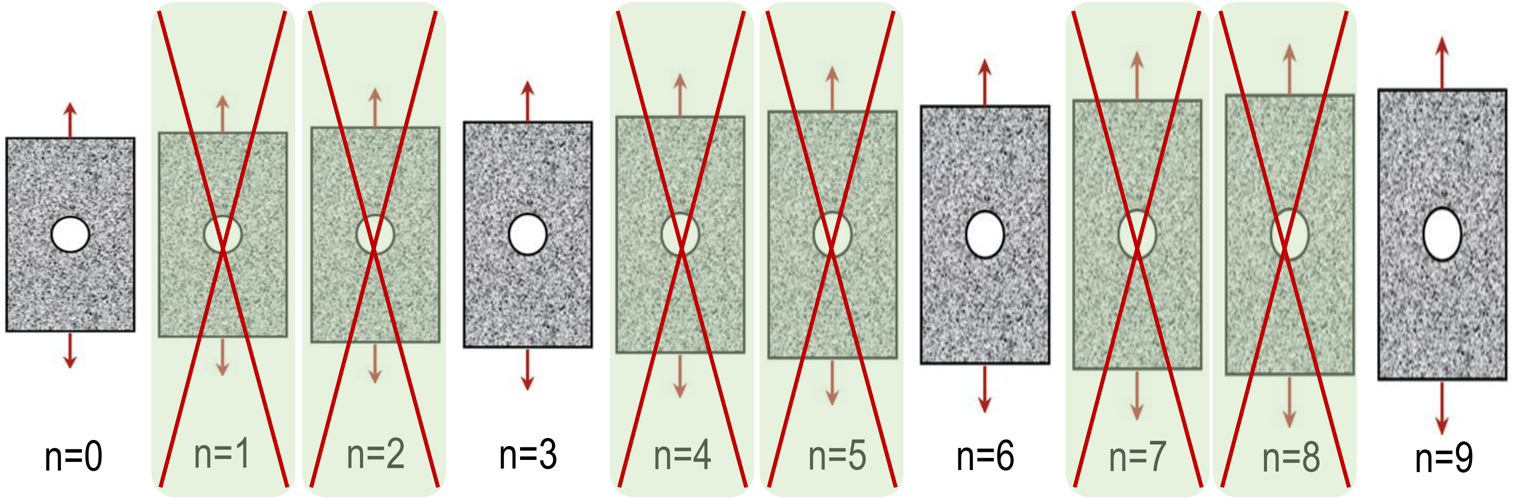

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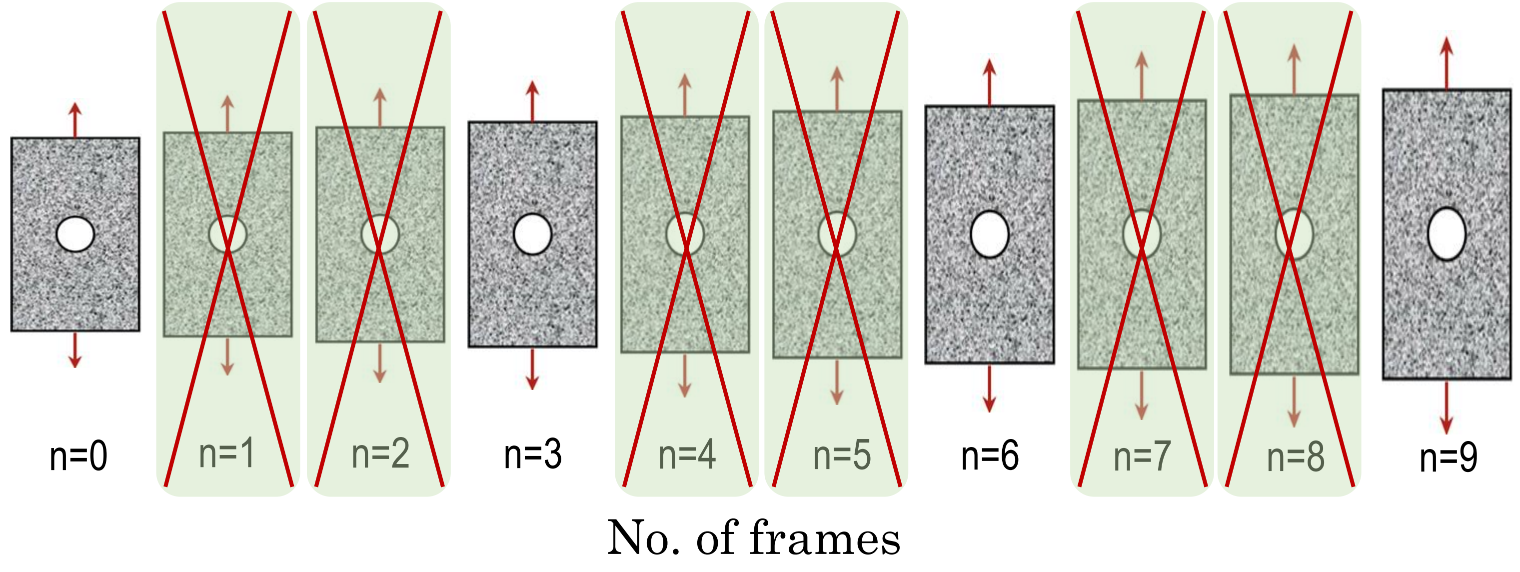
Image acquisition for DIC



Linear elasticity ✓

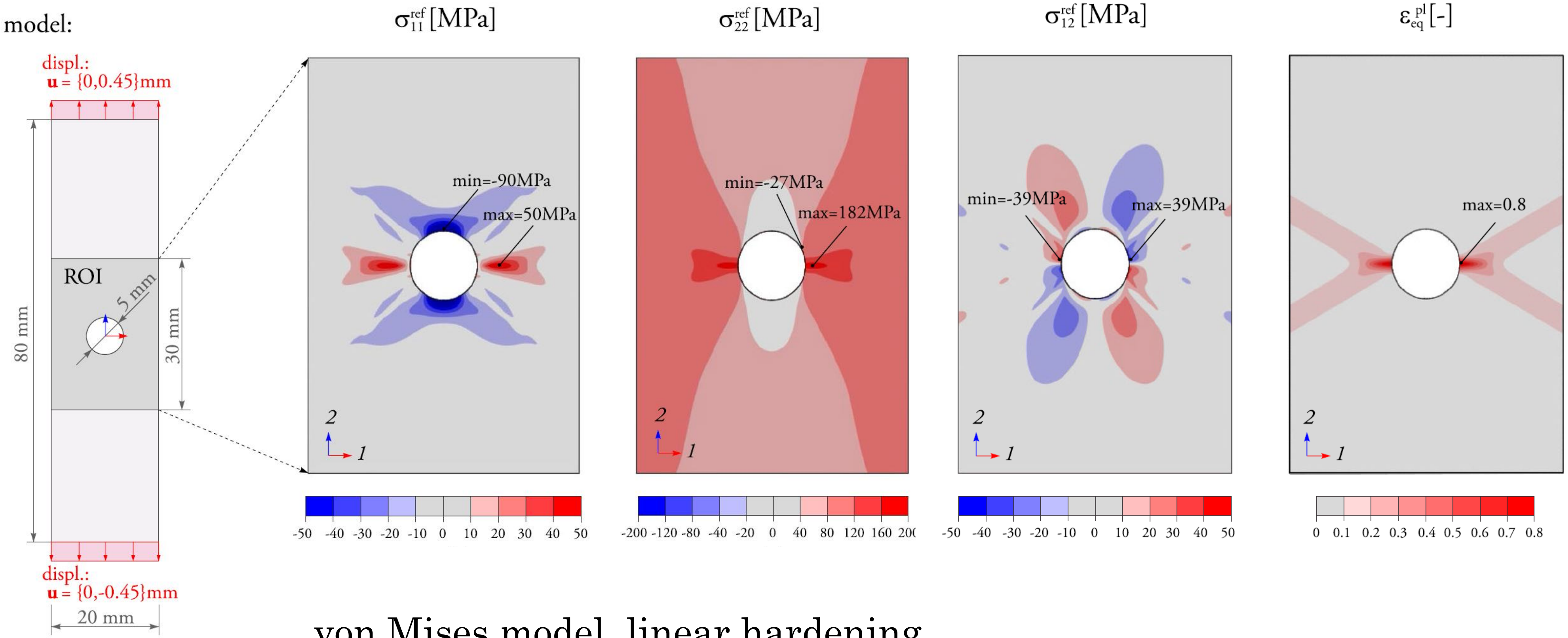
Plasticity ✗

↳ results: path dependent!

The user has control on frame acquisition rate → **improved efficiency?**

Effect of temporal down-sampling

FEA results for the open hole test



von Mises model, linear hardening

More info: Halilovič, Starman, Coppieters: Computationally Efficient Stress Reconstruction from Full-field Strain Measurements, Computational Mechanics, 2024

Effect of temporal down-sampling

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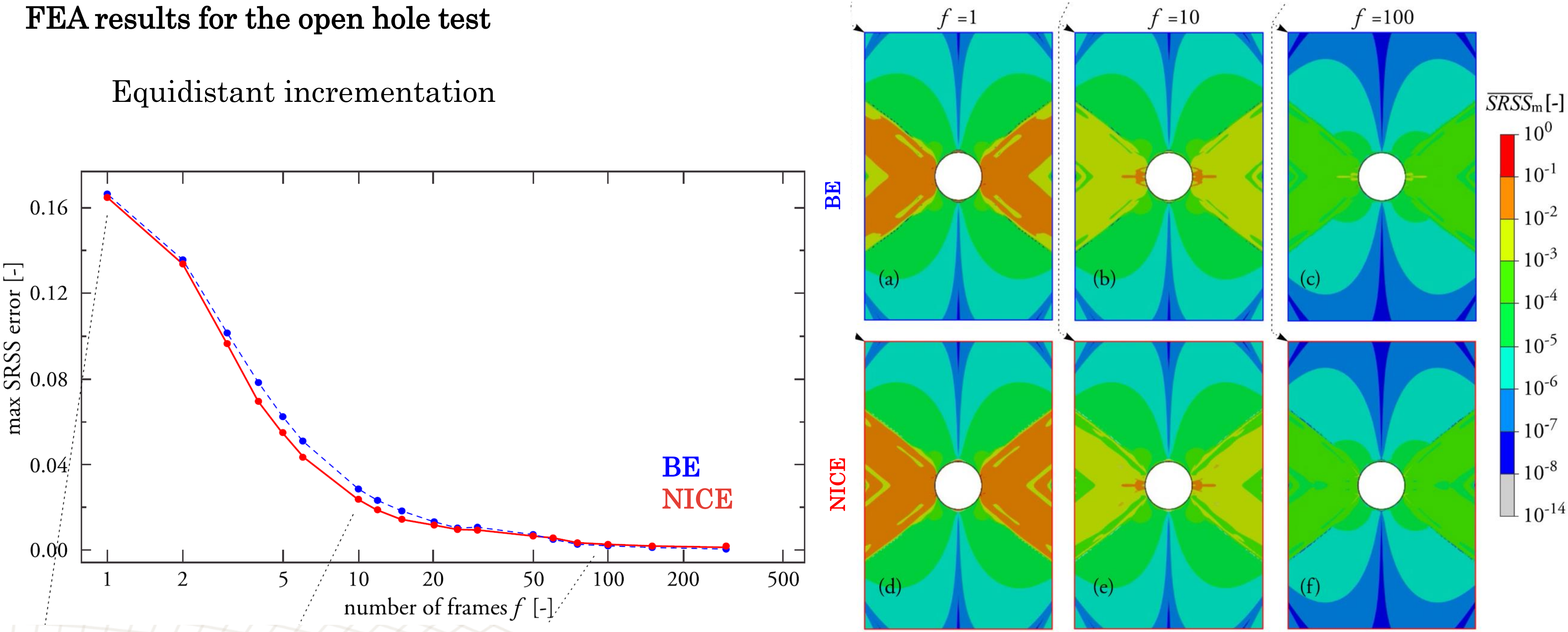
FEA results for the open hole test

Equidistant incrementation



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Effect of temporal down-sampling

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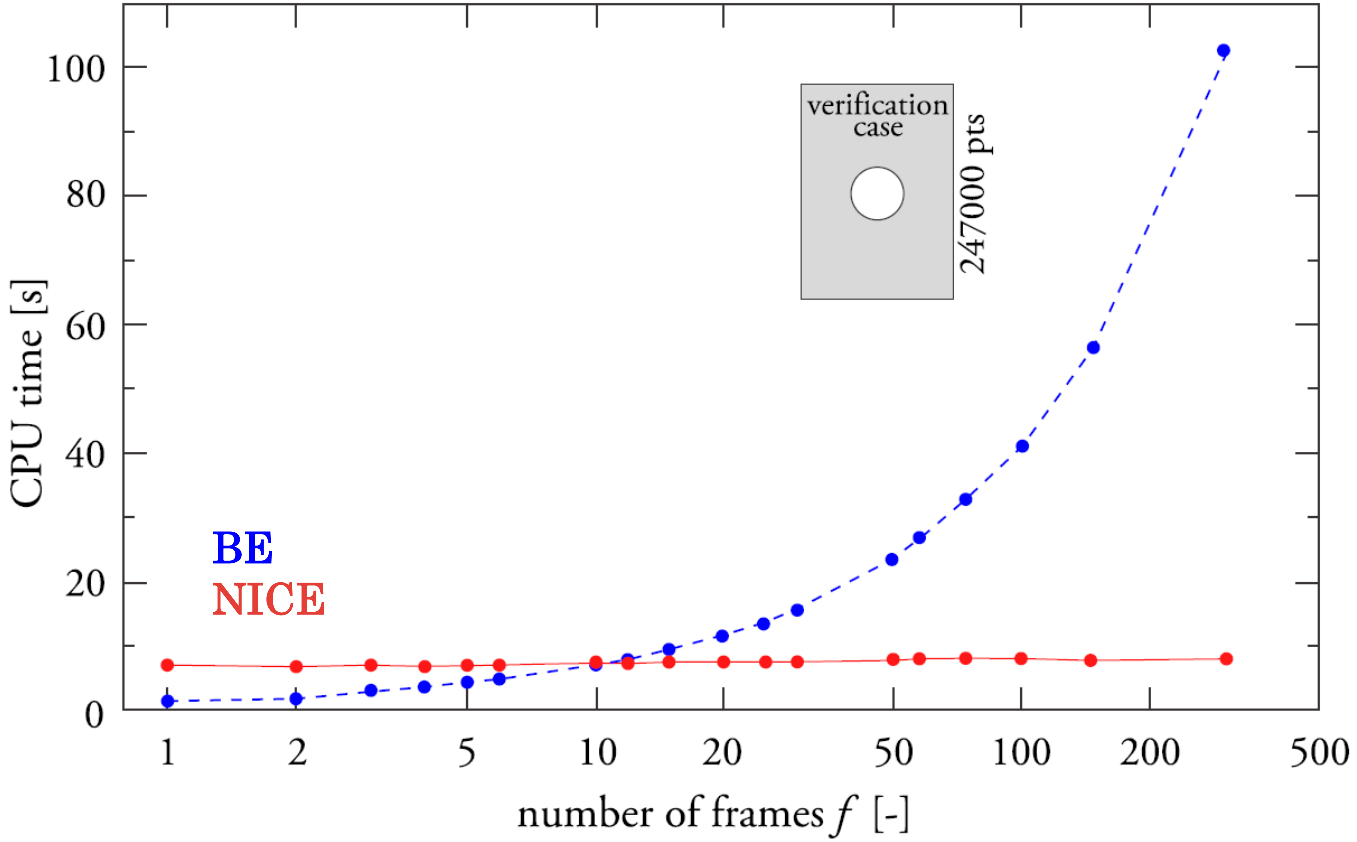
Stress reconstruction efficiency



Theoretically: using NICE, there is no need for down-sampling to save CPU time.

More info: Halilovič, Starman, Coppieters: Computationally Efficient Stress Reconstruction from Full-field Strain Measurements, Computational Mechanics, 2024

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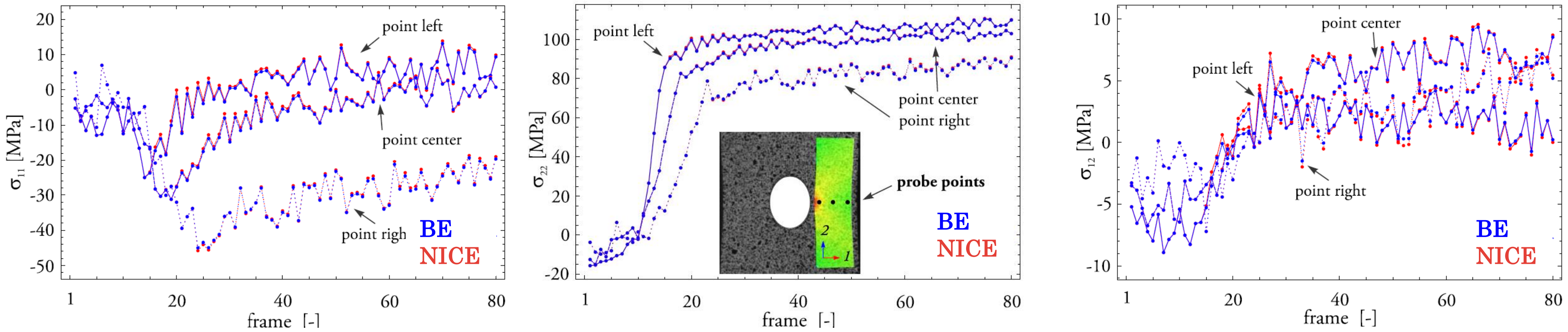
The effect of strain reconstruction on stress integration



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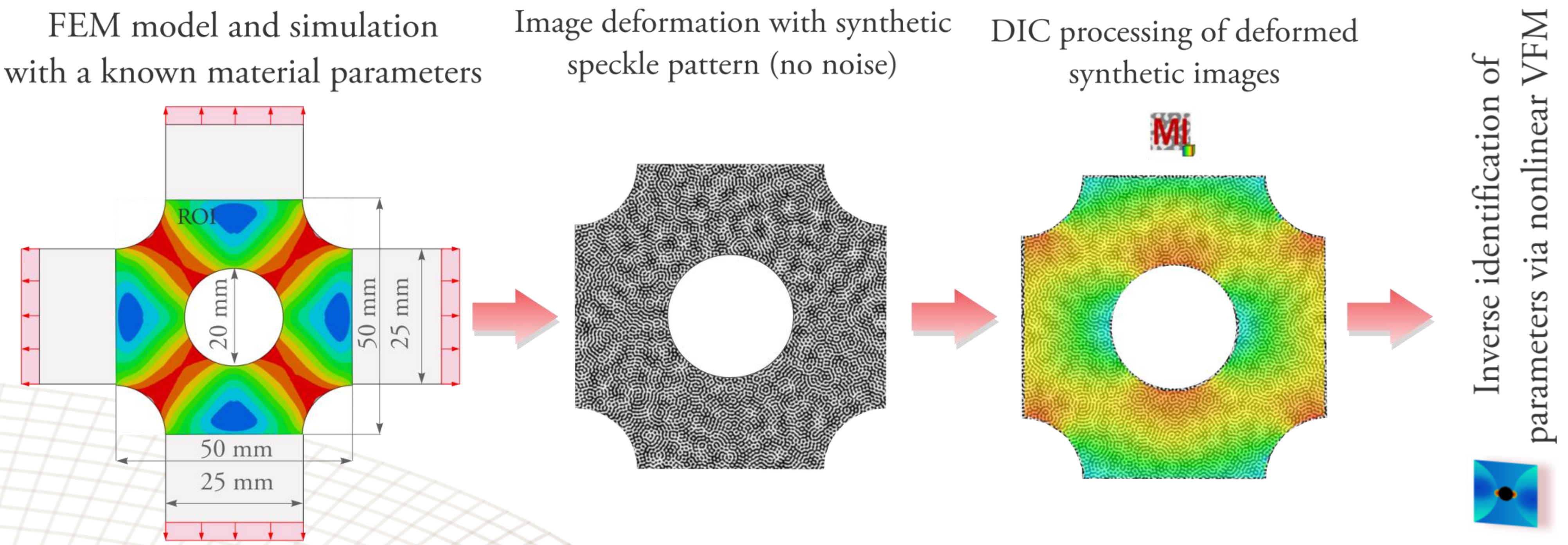
GOAL: Implementation of NICE into MatchID via CMAT module

- identification of plastic anisotropy using YLD2000-2d model parameters using perforated biaxial test
- monitor efficiency, accuracy and robustness

- Material model: Swift hardening + YLD2000-2d
- Identification of material parameters:
- α1, α2, α3, α4, α5, α6, α7, α8, m
- Virtual experiment via biaxial test



Application to VFM

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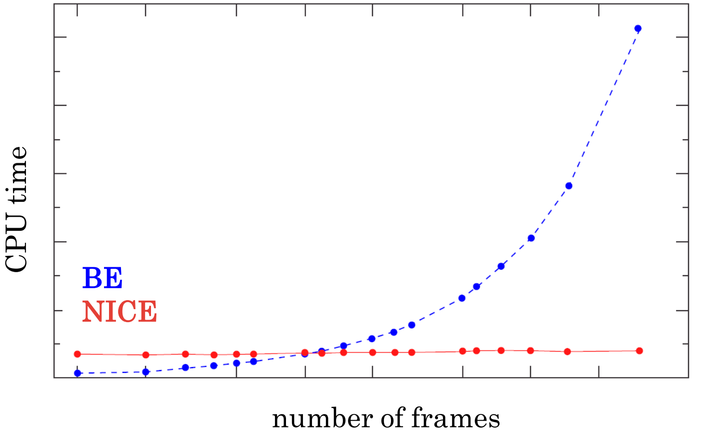

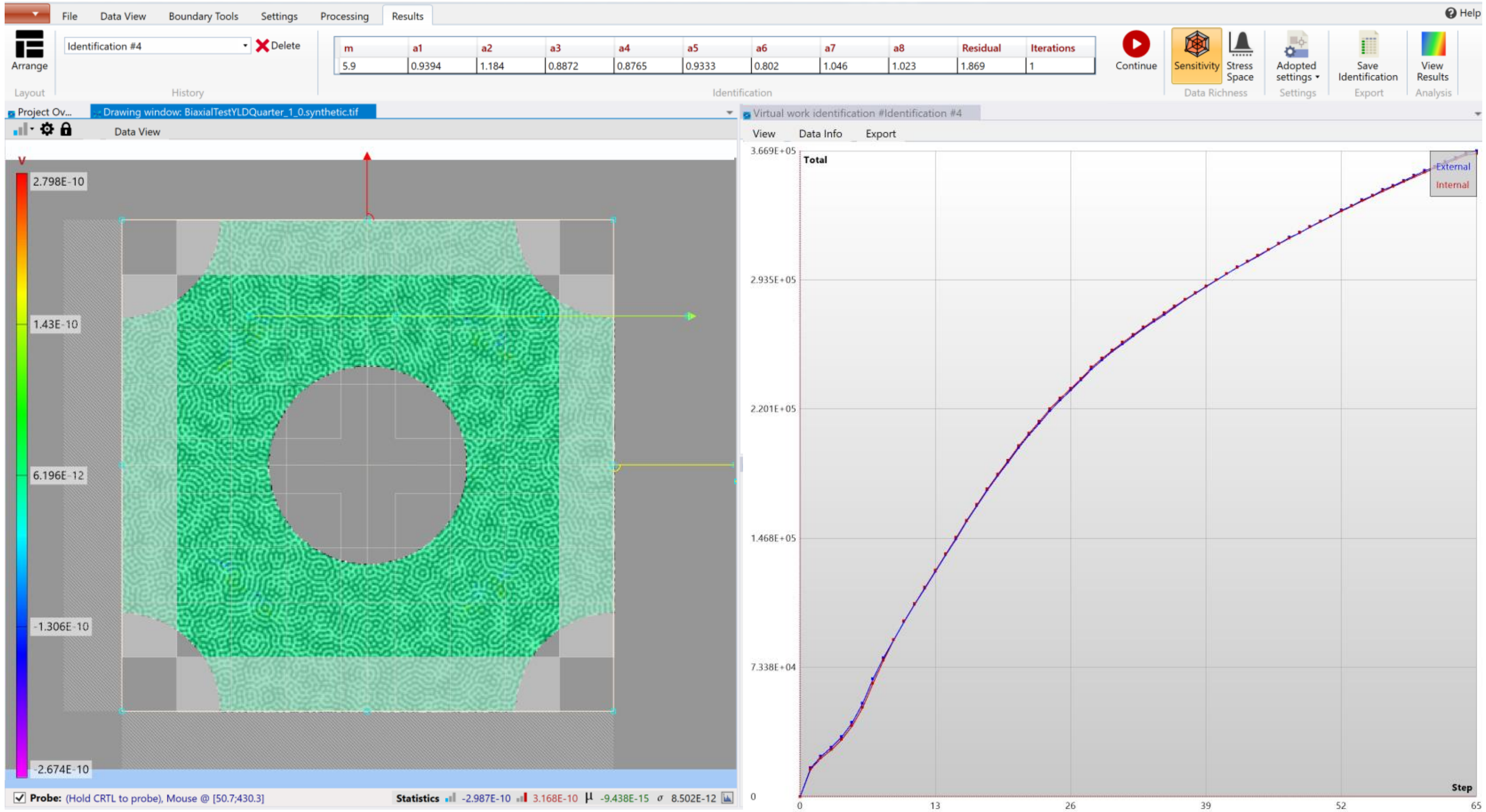

Efficiency: the NICE algorithm is faster in comparison to BE, depending on a number of frames, parameters and iterations

Conclusions

Findings:

- In plasticity, temporal down-sampling of the DIC results increases inaccuracy of results
- The NICE method enables efficient stress reconstruction for highly nonlinear problems
- Using NICE, there is no reason to perform down-sampling to save computational time
- The NICE algorithm was successfully implemented into MatchID via CMAT, and proven to be computationally efficient
- Using NICE computational efficiency can be increased → larger the number of stress reconstructions, larger the benefit of NICE against BE

More info:
Halilovič, Starman, Coppieters: Computationally Efficient Stress Reconstruction from Full-field Strain Measurements, Computational Mechanics, 2024

Thank you for your attention