Coupling the Virtual Fields Method with ANNs for Implicit Constitutive Modelling

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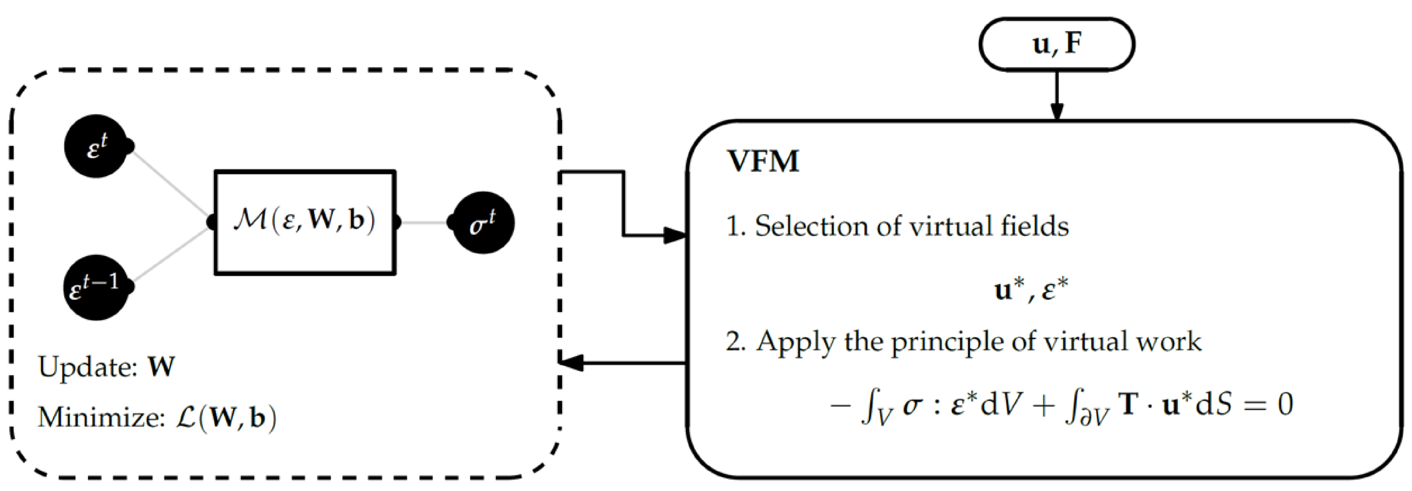
Abstract

Training an ANN for implicit constitutive modelling mostly relies on paired data, usually stress-strain. However, stress is not measurable in a real experiment. As such, training is carried out indirectly using only measurable variables, such as displacements and global force. In this work, displacements and force data are used to train an ANN to predict the stress state of a material. A numerical experiment was created to obtain displacement and global force data for different load distributions. The strain from two subsequent time increments are used as inputs for the ANN to predict the current state of stress. Training is carried out without stress labels to compute the loss. Instead, the local and global equilibrium conditions, corresponding to the application of the Virtual Fields Method (VFM) to the physical model, are employed to compute the loss and update the network parameters, until the predicted stress state is accurate.

Coupled VFM-ANN model

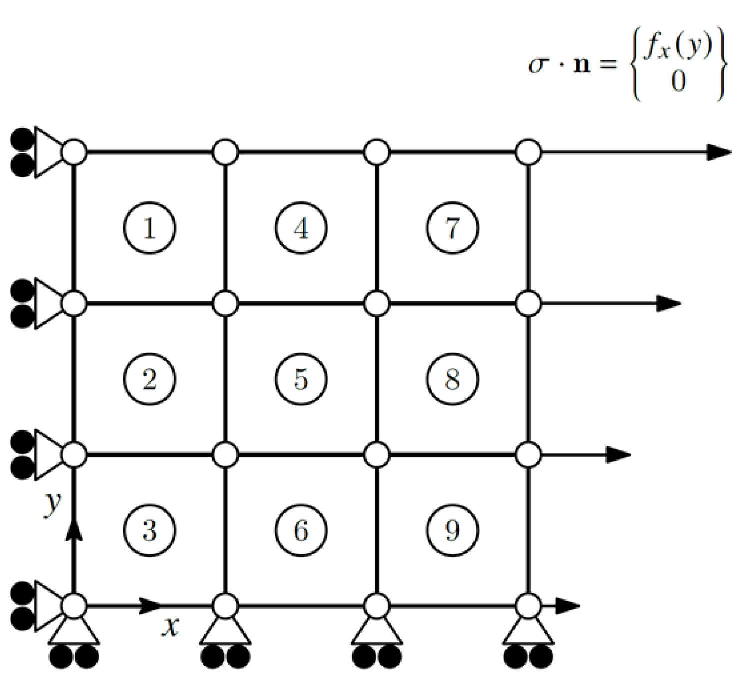
The key elements behind the VFM are the Principle of Virtual Work (PVW) and the choice of virtual fields. According to the PVW, the internal virtual work must be equal to the external virtual work performed by the external forces. The virtual fields are test functions that work as weights and are defined independently of the measured displacements/strains. The chosen virtual fields should be kinematically admissible and should be constant along the boundary where the force is applied.



$$\mathcal{L} = \frac{1}{n_V}\sum_{i=1}^{n_V}\Big(\underbrace{-\int_V \hat{\boldsymbol{\sigma}} : \boldsymbol{\varepsilon}^* \mathrm{d}V}_{\text{internal work}} + \underbrace{\int_{\partial V} \mathbf{T}\cdot\mathbf{u}^* \mathrm{d}S}_{\text{external work}}\Big)^2$$

Data generation

Solid 3×3 mm^2 plate with thickness t = 0.1 mm with the domain meshed with 9 four-node bilinear plane stress elements. Symmetry conditions applied to the boundaries at x = 0 and y = 0 and a surface traction applied to the boundary at x = 3 mm, following a non-uniform distribution varying linearly in the y-direction.



Surface traction distribution

$$f_x(y) = mx + b$$

Global force

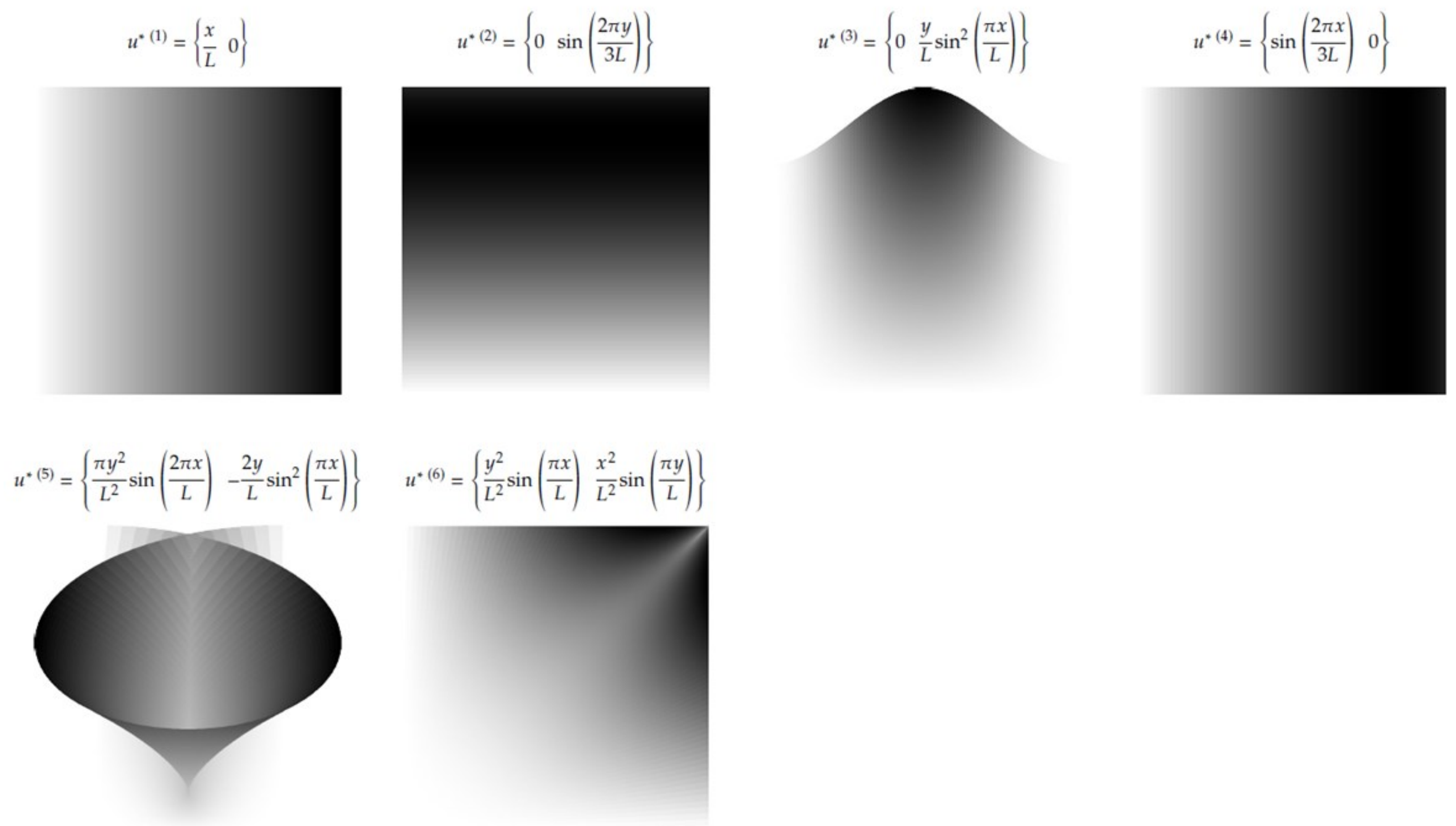
$$FL = \sum_{i=1}^{n} \boldsymbol{\sigma}_i A_i t$$

Swift's hardening law

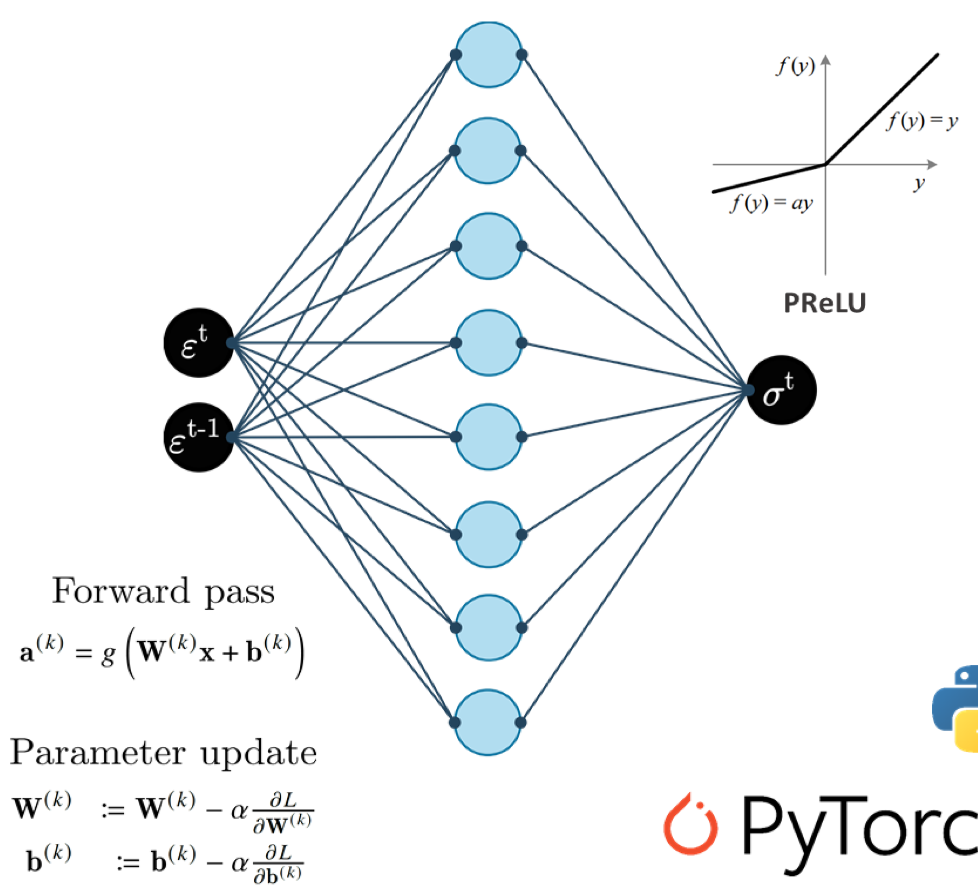
$$\sigma_y = K(\varepsilon_0 + \bar{\varepsilon}^{\mathrm{p}})^n$$

E	ν	σ_0	K	n
210 GPa	0.3	160 MPa	565 MPa	0.26

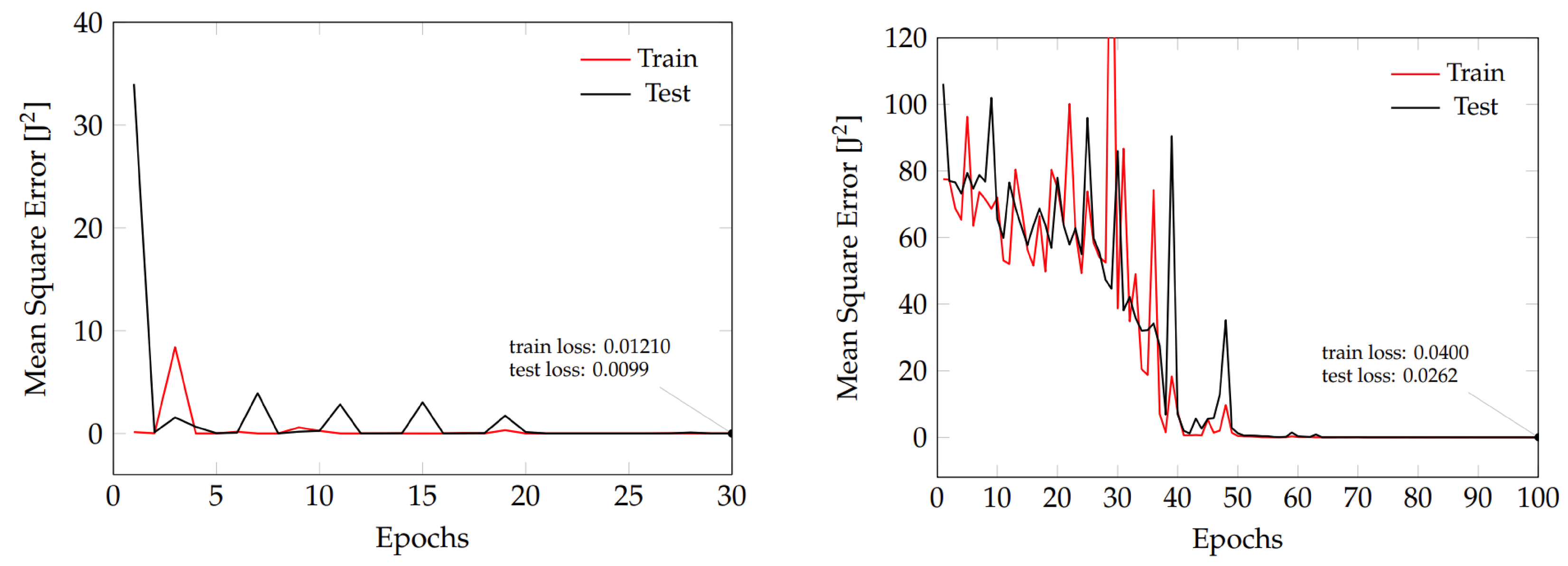
Set of virtual fields



ANN training

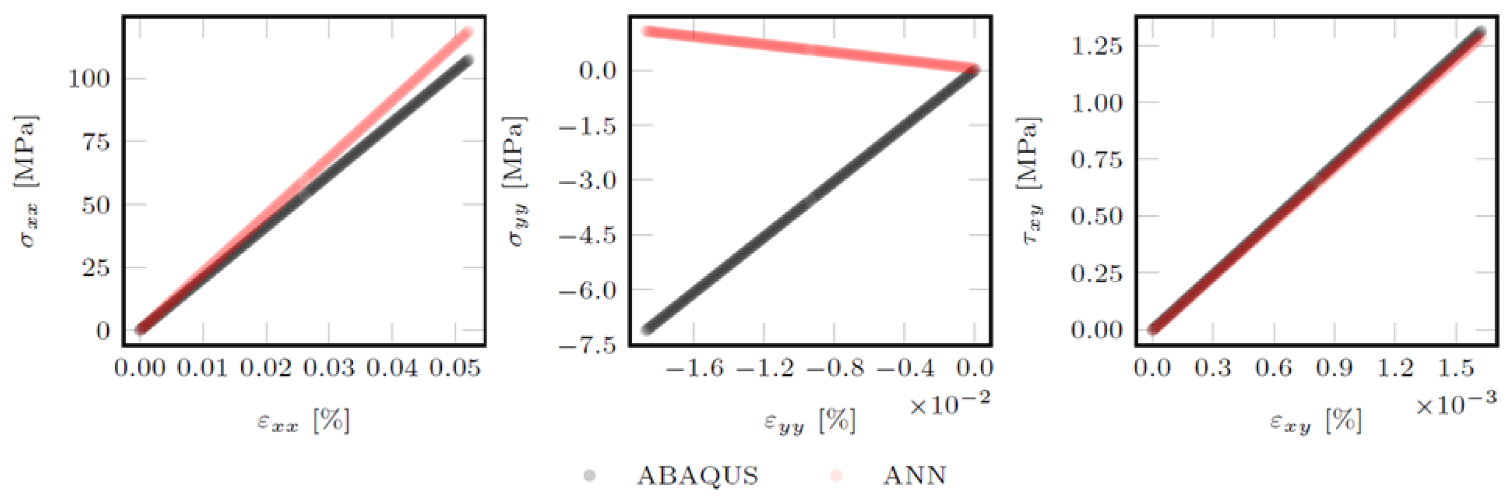


PyTorch

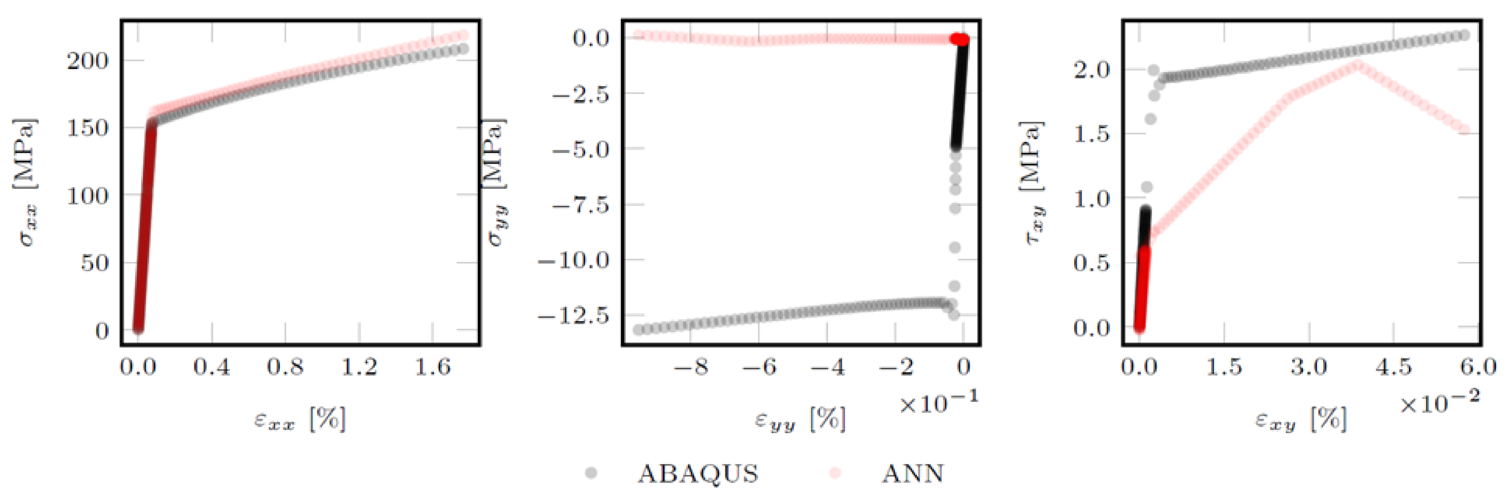


Learning curves for (left) the elastic and (right) the elasto-plastic response models

Results



(c) Elastic model - element #9: $m = 12$, $b = 100$



(d) Elastoplastic model - element #9: $m = 12$, $b = 200$

Conclusions

- A coupled VFM-ANN system was proposed for implicit constitutive modelling
- Elastic and elastoplastic models trained using manually-defined virtual fields
- In general, model predictions were good for stress along x-direction
- Poor stress predictions along y and xy-directions possibly related:
  - manually-defined VFs – tied to the user experience
  - unconstrained optimization
  - mechanical test not rich enough

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