A relative methodology for the uncertainty quantification of material model calibration using a DIC-levelled FEMU and a heterogeneous test

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Introduction and Framework

Non-homogeneous mechanical tests are experiments that present strain heterogeneity regarding the observed different strain states. These came to replace the conventional approaches for mechanical characterization and constitutive model parameter identification. Using Digital Image Correlation (DIC) and the Finite Element Model Updating (FEMU) method, it is possible to calibrate a constitutive model with this type of test. Yet, it is of upmost importance to estimate the errors and uncertainties of these identifications in order to obtain accurate simulations. The DIC procedure alone presents a vast number of sources of error that were already evaluated in [1]. Also, the material parameter identification itself deals with error propagation since it generally compares experimental data measured using optical techniques with numerical data [2]. When using optical cameras to measure the displacement fields, the image noise can be related to displacement uncertainties and the latter to material parameter fluctuations [3]. The FEMU approach is largely used to identify parameters, but the majority of the studies do not account for uncertainty measurements in parameter identification [4,5].The present work aims at evaluating the uncertainties in material parameter identification using a heterogeneous strain test. The general probabilistic and non-probabilistic approaches to quantify errors require large computational efforts that are drastically increased when dealing with model calibration. Thus, an expedited method is essential. Considering the FEMU approach as an optimisation problem and when using a virtual experiment, the global minimum is known, then it is possible to use the KKT conditions [6] for a fast and relative error estimation. The regarded identification approach is a FEMU with a DIC-levelling technique [7], which was already employed with a heterogeneous test in [8]. This technique compares both the experimental and numerical data, similarly, minimising the errors between the two. The mechanical test that feeds the FEMU approach is a strain heterogeneous test proposed in [9], which was originated using shape optimisation. The material under analysis was modelled with the Swift law and Yld2000-2d yield function. These constitutive models are also the target of the uncertainty quantification of the calibration.

Methodology and Implementation

The model calibration with a larger number of material parameters is an optimisation problem that requires severe computational efforts. Its uncertainty quantification is a very time-consuming task. Thus, a simplification of the problem is necessary to quantify its uncertainties. The identification of the parameters is a non-linear unconstrained optimisation problem, which solution satisfies the KKT necessary conditions [6]. This means that, for a cost function φ and the optimum solution $\boldsymbol{\chi}^*$ of the problem, the KKT condition has to be satisfied as follows $\nabla\varphi(\boldsymbol{\chi}^*) = 0$. In theory, for a continuous φ without the presence of noise, these conditions are perfectly verified. Yet, when φ shows a noisy behaviour, the gradient can be different from zero. Figure 1 displays these explanations for a simpler example. This way, the levels of uncertainty can be measured by the deviation to the KKT conditions. Assuming that the condition is verified, the levels of uncertainty due to a specific source of errors $\boldsymbol{\theta}$ can be estimated using the derivative-based local method, written as ${}^{d}/_{d\boldsymbol{\theta}}\left({}^{d\varphi}/_{d\boldsymbol{\chi}}\right)$. Even though not yet demonstrated as an accurate method for uncertainty analysis, it can be used as a relative approach and as a support for fast engineering decision making.

In this work, the function φ that is analysed is the FEMU cost function and $\boldsymbol{\chi}$ is a vector with the Swift law and Yld2000-2d parameters. The investigated sources of uncertainty were the subset, step and strain window sizes that were used for the strain calculation in MatchID, after conducting a FE analysis in Abaqus. A perturbation of 20% was employed to each studied DIC parameter and the derivatives assessed. For a detailed explanation it is recommended to read [10].

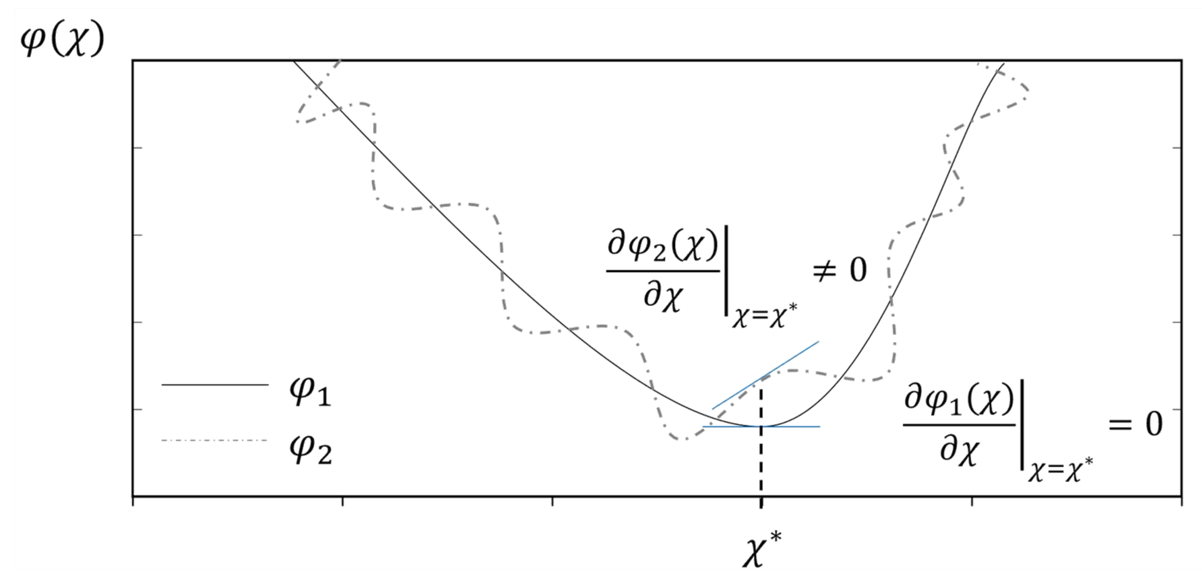

Figure 1. Representation of the KKT conditions in theory (φ_1) and with a noisy function (φ_2).

Results and Analysis

The calculations were completed according to the presented methodology and with a perturbation of 1% in the material parameters. Figure 2 shows the absolute value of the partial derivatives of the cost function with respect to the material parameters in two sets of material parameters. The first is the optimum set of material parameters. Theoretically, its derivatives should be zero, satisfying the KKT conditions. Yet, practically, some of the derivatives are larger than zero. This quantifies the inherent errors caused by the interpolation, transformations, filters and noise of the simulation process that should be accounted for in the interpretation of the following results. The second set of material parameters is obtained randomly, and it is presented to understand if each parameter influences the identification process. This is because, when starting a FEMU identification, the initial set of material parameters usually is different from the optimum set and each parameter should show an influence on the cost function for the optimisation process to flow. If a material parameter is not activated in the mechanical test, the value of $\left|{}^{d\varphi}/_{d\chi}\right|$ would be zero and the methodology present could not lead to any conclusions. Yet, it can be observed that all the parameters are activated and, therefore, influence the identification process. The influence of each source of uncertainty in the problem can be seen in Figure 3. It is observed that α_7 and α_8 are more sensitive to uncertainties. This can be both from the propagation of errors that are observed in Figure 2 and from the influence of each studied source of uncertainty. Also, the step size is the investigated source of uncertainty that largest influences the FEMU DIC-levelled parameter identification, whereas the subset and strain window sizes have a similar impact.

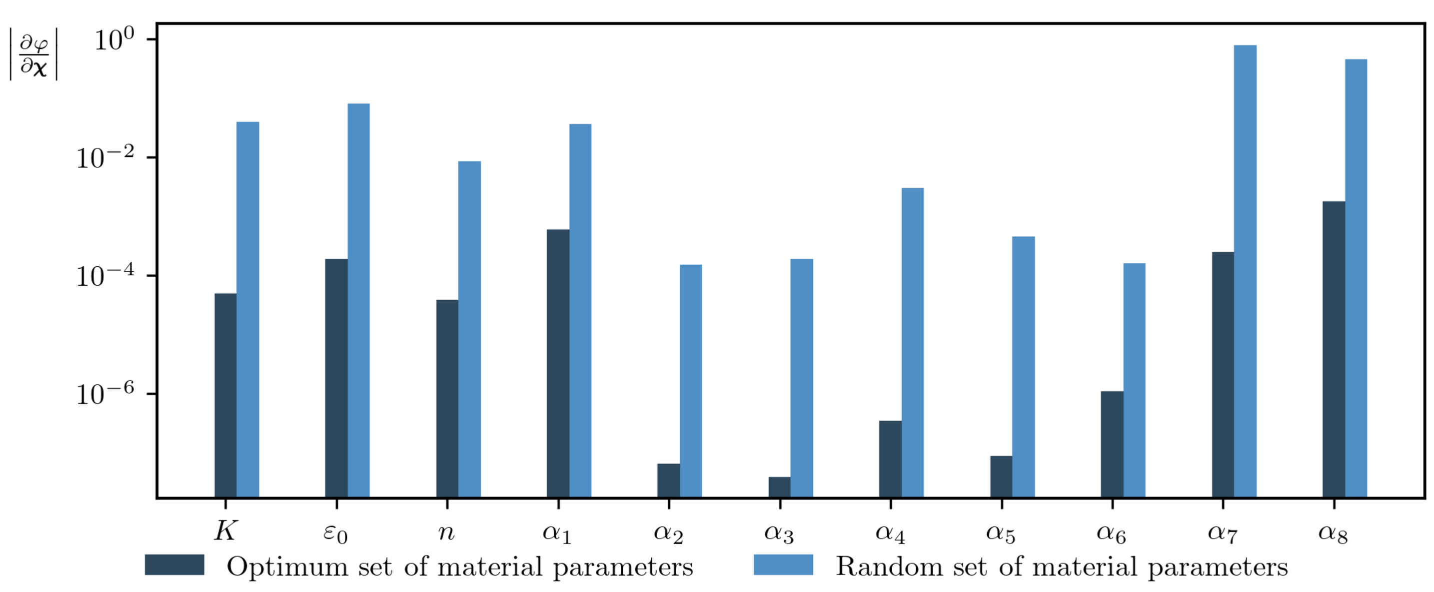

Figure 2. Absolute value of the partial derivative of the cost function φ with respect to the material parameter χ in the optimum and random sets of parameters.

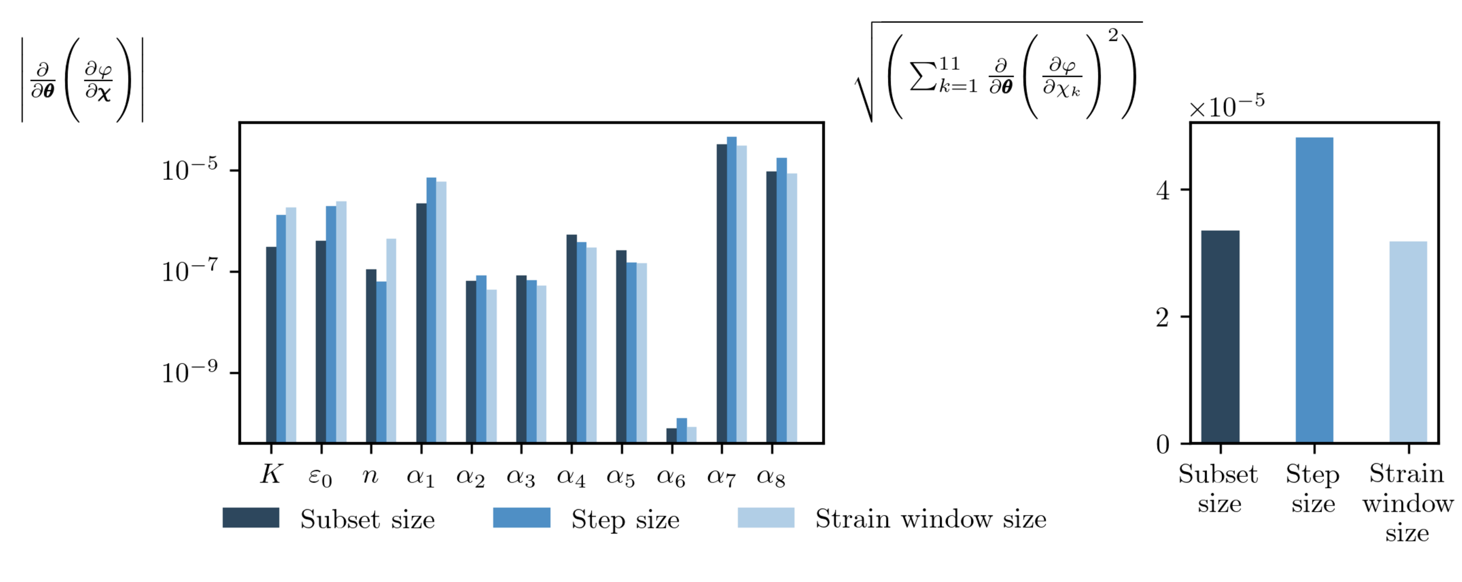

Figure 3. Influence of the subset, step and strain window sizes in each material parameter and in the overall parameter identification approach.

Main Conclusions

In the present work, a methodology was developed to relatively quantify the uncertainties in material parameter identification, using the KKT necessary conditions for non-linear unconstrained optimisation problems. This way, it was possible to simplify the identification problem and estimate the uncertainties with less computational effort, making possible a faster engineering decision making. The calibration problem considered was the FEMU DIC-levelled approach and the uncertainty technique was the derivative-based local method using the finite difference method. The mechanical test used in the identification process was an interior notched specimen for uniaxial loading conditions that presents several strain and stress states simultaneously. The calibration of the Swift hardening law and Yld2000-2d yield function was evaluated. The analysed sources of uncertainty were the subset, step and strain window sizes. It was concluded that the obtained errors in this material parameter identification were originated from the propagation of errors of the measurements, interpolation and discretization of the problem itself and the influence of the sources of uncertainty. Furthermore, the step size has a larger impact on the identified parameters, when compared to the subset and strain window sizes for the investigated speckle pattern and strain gradients. It is recommended to pay special attention when choosing the DIC settings for an inverse identification of material parameters since they have a direct effect on the measured strain gradients and a different impact on the inverse identification of material parameters.

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