

# On the constraints and consistency in implicit elastoplastic constitutive modelling using ANNs and indirect training

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**Abstract:** The training of an Artificial Neural Network (ANN) for implicit constitutive modelling mostly relies on labelled data pairs, however, some variables cannot be physically measured in real experiments. As such, the training should preferably be carried out indirectly, making use of experimentally measurable variables. The unconstrained training of an ANN's parameters often leads to spurious responses that do not comply with the physics of the problem. Applying constraints during training ensures not only the physical meaning of the ANN predictions but also potentially increases the convergence to a global minimum, while improving the model's performance. An ANN material model is trained using a novel indirect approach, where the local and global equilibrium conditions are ensured employing the Virtual Fields Method (VFM). Examples of physical constraints are analyzed and applied during the training process.

**Keywords:** Constitutive model, Elastoplasticity, Neural network, Indirect training, Constrained optimization

## 1 Introduction

ANNs are powerful function approximators that can be used to learn constitutive relationships directly from data [1]. In general, training relies on labelled data pairs, usually stress-strain, obtained from numerically generated datasets. Nevertheless, in a real experiment certain variables (e.g. stress) are not measurable, thus the training should be carried out indirectly using experimentally measurable variables only.

A standard ANN is a black-box model in which its structure is not easily interpretable and there is no guarantee that its predictions are usable, as they can violate fundamental laws of mechanics and thermodynamics [1]. Therefore, it is important to enforce physics-based constraints when using ANNs for implicit constitutive modelling, similarly to Physical Informed Neural Networks (PINNs). These constraints act as a regularization agent for

ANNs, reducing the space of admissible solutions and allowing the network to learn with smaller datasets [2].

The VFM, first introduced by Grédiac [3], is a state-of-the-art method employed in the identification of constitutive parameters. The key elements behind the VFM are the Principle of Virtual Work (PVW) and the choice of virtual fields. According to the PVW, the internal virtual work must be equal to the external virtual work performed by the external forces [4]. The virtual fields consist of virtual strains,  $\varepsilon^*$ , and virtual displacements,  $\mathbf{u}^*$ , defined independently of the measured displacements/strains.

In the present work, ANN models are used to learn constitutive behavior of a virtual material. A novel indirect training methodology employing the sensitivity-based Virtual Field Method (VFM) [4] is used to train the models and study the influence of applying constraints during training.

## 2 Disclaimer

The results reflect only the authors' view, and the European Commission is not responsible for any use that may be made of the information it contains.

## 3 Acknowledgements

Rúben Lourenço acknowledges the Portuguese Foundation for Science and Technology (FCT) for support from grant 2020.05279.BD, co-financed by the European Social Fund, through the programme CENTRO 2020. The authors also acknowledge the financial support of FCT under the projects UID/EMS/00481/2013-FCT through CENTRO-01-0145-FEDER-022083. This project is also supported by the Research Fund for Coal and Steel under grant agreement No 888153.

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