On the inverse identification methods for forming plasticity models using full-field measurements

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Abstract. The simulation of deep drawing processes and its quality is intrinsically dependent on the accuracy of the constitutive model in reproducing the mechanical behaviour of the sheet metal material. Today, the calibration of elastoplastic models – correspondent to the inverse identification of the material parameters – often uses full-field measurements, through Digital Image Correlation (DIC) techniques, to capture non-homogeneous strain fields and states, coupled with non-straightforward numerical inverse methodologies. In the last decade, new parameter identification methodologies, such as the Finite Element Model Updating (FEMU), the Constitutive Equation Gap (CEG) method, the Equilibrium Gap Method (EGM) and the Virtual Fields Method (VFM) have been developed and have proven to be effective for non-linear plasticity models. Nonetheless, the FEMU and the VFM have distinguished themselves from the others. More recently, supervised Machine Learning (ML) techniques have been also used as an inverse identification method. These artificial intelligence-based methods use large datasets of numerical tests to train an inverse model in which the input is the history of the strain field and loads during the test, and the output are directly the material parameters.

The goal of this paper is to analyse, compare and discuss these inverse identification methods, with particular focus on the FEMU, VFM, and ML methodologies. A heterogeneous tensile-load test is considered to compare in detail the FEMU, VFM, and ML strategies.

1. Introduction

Nowadays, modelling and simulation have become a critical stage in product design. Even in the development of new materials, engineers resort to simulation for fast progress and comprehension of the intrinsic phenomena. However, in Solid Mechanics, the simulation is fully dependent on the numerical methods and the material models. Although the numerical methods and their approximations can be evaluated using benchmarks and convergence analysis (e.g. numerical mesh analysis), the material models can only be evaluated when confronted with experimental tests. Yet, each classical experimental test represents a simple phenomenon and a single stress/strain state, making a rigorous experimental campaign very laborious. Moreover, complex states as the ones experienced in deep drawing are not represented and taken into account in these experimental campaigns, therefore a correct material behaviour characterisation is not guaranteed.

Recently, experimental full-field measurements, particularly using Digital Image Correlation (DIC) techniques, opened large potentialities in material characterization in the sense that experimentally full-field displacements and strain maps can be measured. The advantage of local strain analysis of the Finite Element Analysis (FEA) was therefore reduced because these can now be experimentally measured. However, local stress analysis is still is not measurable experimentally, and is only possible to obtain with FEA (called numerical stress reconstruction).

A phenomenological macroscopic material constitutive model, in particular in elastoplasticity, relates the strain with stress. Therefore, using the strain knowledge coming from full-field measurements, the constitutive model computes the stress field, which could be easily calibrated and compared with the experimental stresses, again, if these could be measured. However, during mechanical tests, independently of homogeneous or heterogeneous, only the load (which corresponds to the stress surface integration) can be measured. Therefore, the calibration of a material constitutive model must be done indirectly because the model's output is not locally and (yet) experimentally known. Nevertheless, knowing the model mathematical structure is already a great help[1] since the model calibration reduces to a parameter identification problem.

In the last decades, several strategies have emerged to solve the inverse parameter identification problem in solid mechanics, inclusively, using full-field measurements [23, 11]. Clearly the first and the most known is the Finite Element Model Updating (FEMU) [14] followed by the Virtual Field Method (VFM) [10, 12, 28, 26]. Other methods must be also highlighted, such as the Constitutive Equation Gap Method (CEGM) [19], the Equilibrium Gap Method (EGM) [8, 9], the Constitutive Compatibility Method (CCM) [24], the Dissipation Gap Method [4], the Self-Optimizing Method (Self-OPTIM) [32] and the Integrated Digital Image Correlation Method (Integrated-DIC) [29]. Other methods can be retrieved from a literature review, however, these seem to be variations of the previous methods. From this list, only the FEMU, the VFM, the CEGM and the EGM proved to be effective in the parameter identification of non-linear models [23]. Very recently, with the explosion of data-driven methods, Machine Learning (ML) and Artificial Intelligence (AI) techniques, supervised learning was used to identify the parameters of constitutive models [5, 6, 17].

The distinction between the parameter identification methods is in the use of the strain and load as an inverse method. While the FEMU method uses the numerical strain field and the load, and directly and iteratively compares them to the correspondent experimental values, other methods such as the VFM use equilibrium equations to guide the identification process. In these equilibrium equations, the experimental strain and load are used in both sides of the equilibrium equation, the stress field is reconstructed using the experimental strain field and a residual is minimised. The difference is then:

- For the CEGM, the equilibrium equation is energetic (energy norm) and the objective function to minimise is based on the difference between an updated statically admissible stress field and the stress field reconstructed using the experimental strain field and the constitutive model [19]. The particularity of this method is the statically admissible stress field, which must verify the boundary conditions of the experimental test, as well as the equilibrium equation [23]. An FEA model can be used to calculate this latter stress field.
- For the EGM, a local equilibrium is assessed using the continuity of the stress vector at the interfaces [8, 9]. This local equilibrium can be calculated using finite differences between the elements interfaces, and their gap must be minimised. Here again, the stress update algorithm uses the (measured) experimental strain. For the boundary interfaces, the

[1] Although here the previous knowledge of the mathematical structure of the model is considered an advantage for the material characterisation because it simplifies the identification problem, for a general discussion it can be seen as a disadvantage because this formulation can work as a constraint for the reproduction of the material behaviour.

equilibrium is prescribed using the external load. For the case of a free boundary interface, the load is zero. Therefore, this method minimises the local stress equilibrium into the domain and the boundary to find the parameters.

- The equilibrium in the VFM method is taken from the principle of virtual work, which balances the internal virtual work with the external virtual work in the absence of body-forces [27], i.e. verifies if the stress fields are statically admissible. The external work uses the experimentally measured load and the internal work is found using the stress reconstructed from the experimental strain field and the constitutive model. The VFM has the advantage of producing both local and global equilibrium, depending on the virtual field used. These virtual fields act as filters to obtain different work equilibrium equations and can be obtained manually or automatically. There are no limitations on the number of virtual fields and there is no need for a FEA analysis.

All the previous inverse techniques require an optimization algorithm for an efficient search of the parameters while minimizing its objective functions (equilibrium conditions or external "numerical *vs* experimental" differences).

Machine learning methods, particularly the well-known Artificial Neural Networks (ANN), use data to build a model. These are (data-driven) regression models in which the regressor (or its mathematical formulation) is not initially known and can be found together with its parameters during a fitting operation, which is called supervised training. For the parameter identification problem, the goal is to develop a model that, with the knowledge of the strain field and load[2], can give suitable numerical parameters. This approach makes use of the inverse problem formulation to directly build an inverse model. No stress update algorithm is required nor any equilibrium conditions. However, its black-box characteristic frightens the engineers because there is no guarantee of equilibrium or any other physics. Only data is used and all Key Performance Indicators (KPI) metrics are statistical, as in regressions.

The goal of this paper is to promote a discussion on the parameter identification methods, with particular emphasis on the FEMU, VFM and ML methods, representing the external, equilibrium and data-driven approaches, respectively. For that purpose, the paper is divided into two parts. The first part, which includes sections 2-4, presents the methods, their particular formulations and their implementation. The second part (section 5) presents an example to discuss the performance of each approach.

2. Parameter identification as an inverse problem

In Solid Mechanics, FEA simulation requires the geometry of the solid, boundary conditions including forces, and the material behaviour, which is reproduced by the material constitutive model and its parameters and properties. The solution of an FEA simulation includes the displacements, strains, and stresses. This problem is considered the *direct problem* and its solution is obtained by FEA simulations using the Finite Element Method (FEM). The parameter identification problem is one of the *inverse problems* of the previous direct problem, where the displacement and strain fields become known and the material parameters are unknown. This problematic is illustrated in Figure 1.

In a general formulation, the inverse parameter identification problem can be written as the

[2] It is important to recall that the stress field is not directly available from the mechanical tests. Only the global force can be measured.

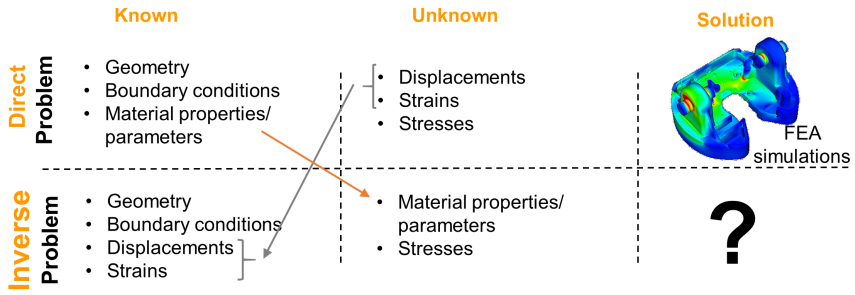

Figure 1. Direct problem versus inverse problem in the material characterisation and parameter identification.

search for the set of the r parameters $\mathbf{A}$ that

$$\begin{aligned} \underset{\mathbf{A}}{\text{minimise}} \quad & \mathcal{L}(\boldsymbol{\sigma}(\mathbf{A}, \boldsymbol{\varepsilon}^{\text{exp}}, \boldsymbol{\varepsilon}^{\text{num}}), \boldsymbol{\varepsilon}^{\text{exp}}, \mathbf{u}^{\text{exp}}, \mathbf{F}^{\text{exp}}, \ldots) & & \\ \text{subject to} \quad & g_m(\mathbf{A}) \leq 0, & & m = 1, \ldots, M \\ & \mathbf{K}(\mathbf{A})\mathbf{u} = \mathbf{F}, & & \\ & A_i^{\min} < A_i < A_i^{\max}, & & i = 1, ..., r \ , \end{aligned} \tag{1}$$

where $\mathcal{L}$ is a function that relates the measured experimental displacements $\mathbf{u}^{\text{exp}}$, strains $\boldsymbol{\varepsilon}^{\text{exp}}$ and loads $\mathbf{F}^{\text{exp}}$ with their numerical correspondent. This function must use the constitutive model to reconstruct the stress, however, the input of the stress update algorithm changes from one method to other. The inverse problem can be subjected to some constraints. Inequality constraints are represented by M functions g and the side constraints depend on the maximum and minimum of each variable, represented by $A^{\max}$ and $A^{\min}$, respectively. An equality constraint $\mathbf{Ku} = \mathbf{F}$ is also defined, however, it is generally satisfied if an FEA is used or other static equilibrium condition. For non-linear constitutive models, which depend on the deformation history, $\mathcal{L}$ must be integrated in (virtual) time, meaning that it must be evaluated for n_{s} time instants:

$$\mathcal{L}(\mathbf{A}) = \sum_{i=1}^{n_{\text{s}}} \mathcal{L}_i(\mathbf{A}). \tag{2}$$

3. Inverse methodologies for full-field measurements

The definition and solving procedure for $\mathcal{L}(\mathbf{A})$ is the main difference of the methods. For external and equilibrium methods, the parameters are found iteratively by $\mathbf{A} = \min \mathcal{L}(\mathbf{A})$. The Machine Learning method build the inverse model $\mathcal{L}^{\text{ML}}$ and obtain directly the parameters solving $\mathbf{A} = \mathcal{L}^{\text{ML}}(\boldsymbol{\varepsilon}^{\text{exp}}, \mathbf{F}^{\text{exp}})$.

3.1. External methods and the Finite Element Model Updating (FEMU)

The external methods are the ones that the experimental values are external to the constitutive model and the stress reconstruction. The numerical calculations are completely separated from the experimental measurements and their comparison is made with a kind of numerical-experimental gap metric. The FEMU is the most widely used method for parameter identification and reportedly first employed by Kavanag and Clough in 1971 [14].

The FEMU method generally uses a FEA model[3] of the mechanical test to calculate the displacements, strain and stress fields with the assumption that the experimental boundary

[3] Although FEA is clearly the most used simulation method in solid mechanics, other numerical methods for space and time integration can be used, such as IGA, finite differences, etc.

conditions are fully reproduced numerically. This assumption is one of the major disadvantages of the FEMU method because some boundary conditions and phenomena are very difficult to reproduce. Contact and friction (resulting in sliding) in the grips is an example of this difficulty. Today, these boundary conditions can be retrieved from the DIC displacement measurements and input in the numerical model, however, the measurement noise can produce stress concentrations at the boundaries.

The stress integration of the constitutive model is made inside the FEA. However, only strains and global loads are compared. For the case of full-field measurement[4], the objective function $\mathcal{L}$ can be written as

$$\mathcal{L}^{\mathrm{FEMU}}(\mathbf{A}) = \sum_{i=1}^{n_\mathrm{s}} \left[\sum_{j=1}^{n_\mathrm{p}} \left(\varepsilon_{xx}^{\mathrm{exp}} - \varepsilon_{xx}^{\mathrm{num}}(\mathbf{A})\right)_j^2 + \left(\varepsilon_{yy}^{\mathrm{exp}} - \varepsilon_{yy}^{\mathrm{num}}(\mathbf{A})\right)_j^2 + \left(\varepsilon_{xy}^{\mathrm{exp}} - \varepsilon_{xy}^{\mathrm{num}}(\mathbf{A})\right)_j^2 \right]_i + \sum_{i=1}^{n_\mathrm{s}} \left(\mathbf{F}^{\mathrm{exp}} - \mathbf{F}^{\mathrm{num}}(\mathbf{A})\right)_i^2 . \quad (3)$$

The xx, yy and xy subscripts denote the strain components at each of the n_p space integration points in a 2D plane stress formulation. The minimization of this function leads to a perfect match on the strain field and on the global load. A scheme of this approach is depicted in Figure 2.

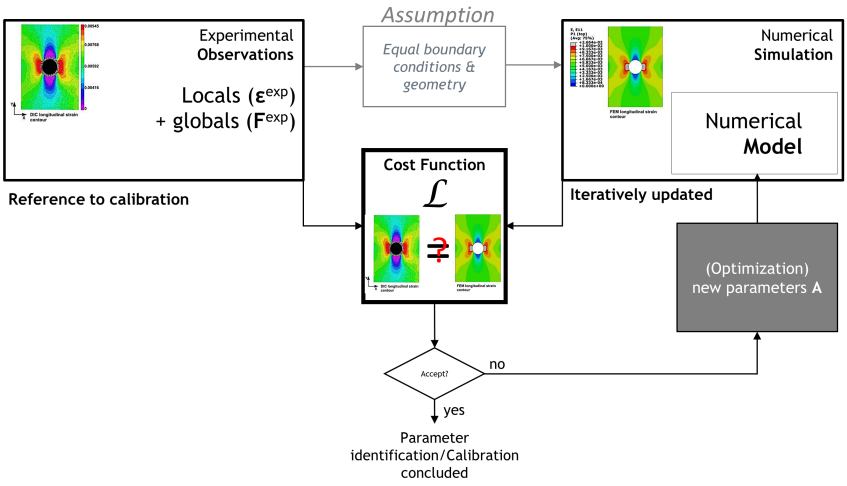

Figure 2. Schematic flowchart for the FEMU method.

One recent discussion concerns strain interpolation. Although there is a consensus for time interpolation, because it is rather easy to use the same time instants, the space interpolation can lead to smoothing or discontinuity errors if the numerical mesh is quite different from the experimental DIC subset mesh. A DIC-leveled approach [18, 13] can be the solution, however, accompanied by a large increase in computational effort.

It is also good practice to normalize the objective function 3 and give equal weight to both the strain and forces. However, generally, the choice of the weights is made intuitively and can lead to unsuccessful results.

[4] One advantage of the FEMU method is that it is easily adapted to the available experimental data and can be used with partial measurements or even with a single average value.

3.2. Equilibrium methods and the virtual field method (VFM)

Equilibrium methods solve the parameter identification problem with the verification of balance/equilibrium conditions. For the case of the VFM, the equilibrium condition comes from the Principle of Virtual Works, which states that, in the absence of acceleration and body forces, the internal virtual work must be equal to the external virtual work, i.e.

$$W^{\star}_{\text{internal}}(\mathbf{A}) = W^{\star}_{\text{external}} \Leftrightarrow \underbrace{\int_{\Omega} \boldsymbol{\sigma}(\mathbf{A}, \boldsymbol{\varepsilon}^{\text{exp}}) : \boldsymbol{\varepsilon}^{\star} dV}_{\text{Internal virtual work}} = \underbrace{\int_{\Gamma_f} \bar{\mathbf{f}}^{\text{exp}} \cdot \mathbf{u}^{\star} dS}_{\text{External virtual work}} , \tag{4}$$

for a body Ω and a surface density of the force $\bar{\mathbf{f}} = \mathbf{F}/S$ applied over the boundary Γ_f. It must be noted that the principle of virtual work is independent of the constitutive model, therefore, it can be widely used. The static equilibrium between the external and internal work is a global equilibrium and requires the full knowledge of all forces, including the reactions of the boundary conditions, which is very difficult to measure. However, the major advantage of this method is the use of a kind of filter functions called virtual fields, defined by the virtual strain $\boldsymbol{\varepsilon}^{\star}$ and displacement $\mathbf{u}^{\star}$, which allow to obtain multiple local and global equilibrium equations and, therefore, filter unknown boundary forces[5]. For more details concerning the demonstration of the VFM, please refer to [28].

Although the major advantage of this method comes from the virtual fields (VFs), these are also its main drawback. The choice of suitable VFs is not straightforward and is a critical stage in the identification process. These VFs must be chosen satisfying (i) zero virtual displacement on the constrained boundary condition, due to the unknown forces on these boundaries, (ii) be constant on the load boundary condition, due to the knowledge of the resultant of the applied force, instead of its distribution, and (iii) C^0 continuity. Today, the drawback of VFs selection was clearly minimized with its calculation by stiffness-based [26, 1] and sensitivity-based [20] procedures. For non-linear elastoplasticity, the objective function is then,

$$\mathcal{L}^{\text{VFM}}(\mathbf{A}) = \sum_{i=1}^{n_{\text{s}}} \left(\int_{\Omega} \boldsymbol{\sigma}(\mathbf{A}, \boldsymbol{\varepsilon}^{\text{exp}}) : \boldsymbol{\varepsilon}^{\star} dV - \int_{\Gamma_f} \bar{\mathbf{f}}^{\text{exp}} \cdot \mathbf{u}^{\star} dS, \right)_i^2 \tag{5}$$

which should be minimised. Figure 3 illustrates this approach.

3.3. Data-driven and Machine Learning (ML) methods

When solving inverse parameter identification problems, data-driven methods are similar to response surface methods in the sense that these use the data to make the relation between the input and output. This relation is built as a regression. Therefore, data-driven methods do not solve an inverse problem but previously build directly the inverse model as a regression. Machine learning methods, where ANNs, support vector machines and random trees are included, are the new competitors for solving the parameter identification problem. This ML approach uses direct problem solutions to fit (here called supervised learning) the regression model. Although this approach does not inversely and iteratively identify the parameters of the constitutive model, it still solves an identification problem but for the hyper-parameters of the ML model. However, this problem is much simpler because the regressor structure (the topology or architecture of the ML model) is well known and can be easily differentiated. After proper learning, the solution for the material model identification is promptly obtained. The disadvantage of this method is the

[5] Although these virtual displacement and strain seem to have the meaning and units of length, in fact, these functions cannot be seen as real displacement and strain. Nevertheless, these remain with the units of displacement and strain. It is the use of virtual quantities that makes the principle of virtual work dissimilar to the energy balance. The VFM grounds on the local equilibrium equations and force boundary equations.

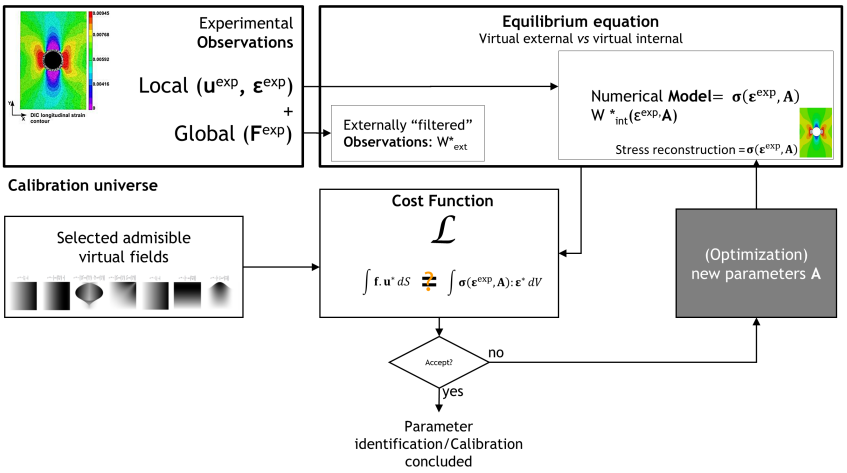

Figure 3. Schematic flowchart for the VFM method.

computational effort to build the direct problem database. For numerical material constitutive models, which are going to be used in FEA, the database can be built using the same FEA simulator considering the strain field and the load as input feature and the parameters as output features. This method is illustrated in Figure 4.

The formulation for a ML approach can be written as

$$\mathbf{A} = \mathcal{L}^{\mathrm{ML}}(\boldsymbol{\varepsilon}^{\mathrm{exp}}, \mathbf{F}^{\mathrm{exp}}, \mathbf{W}, \mathbf{b}), \tag{6}$$

where

$$(\mathbf{W}, \mathbf{b}) = \min \mathcal{F}\left(\mathbf{A}_i, \boldsymbol{\varepsilon}_i, \mathbf{F}_i\right)_{\mathrm{FEA},i=1,\ldots,ndata}. \tag{7}$$

The hyper-parameters $\mathbf{W}$ and $\mathbf{b}$ of the activation functions[6] are found by minimising a given loss function $\mathcal{F}$, which is generally the mean square error (MSE) of all *ndata* residuals with a regularization term. The training database $(\mathbf{A}_i, \boldsymbol{\varepsilon}_i, \mathbf{F}_i)_{\mathrm{FEA},i=1,\ldots,ndata}$ is used once for the training, then the same ML model architecture with the found hyper-parameters can be used for different experimental sets $(\boldsymbol{\varepsilon}^{\mathrm{exp}}, \mathbf{F}^{\mathrm{exp}})$. However, it is essential that the experimental input set has the same number and, for the strain field, location of the set $(\boldsymbol{\varepsilon}_i, \mathbf{F}_i)_{\mathrm{FEA},i=1,\ldots,ndata}$ used for training. A DIC-levelled approach can be also used to create the training database. Here, similarly to the FEMU approach, the experimental geometry, loading, and boundary conditions must be very well reproduced by the FEA simulation used for training.

4. The role and dependency on the optimization technique

All procedures for inverse parameter identification require optimization techniques and all formulations lead to least-square optimization problems (see equations 3,5 and 7). Therefore, optimization algorithms such as Levenberg-Marquardt [21] or Trust Region Reflective algorithms are expected to be used. However, these are gradient-based methods and, consequently, are sensitive to the objective function behaviour and noise, and the initial set of parameters. Alternatives to gradient-based least square algorithms are derivative-free least-square algorithms. Nonetheless, unexpectedly, this type of algorithms is not used by the scientific community. Instead, the Nelder-Mead is commonly used in the field. An analysis concerning some optimization algorithms can be seen in [25], however, this is still an open question.

[6] These activation functions depend on the ML model. In the case of ANN, these functions are related to neurons. However, for random trees or support vector machines, these have different meanings.

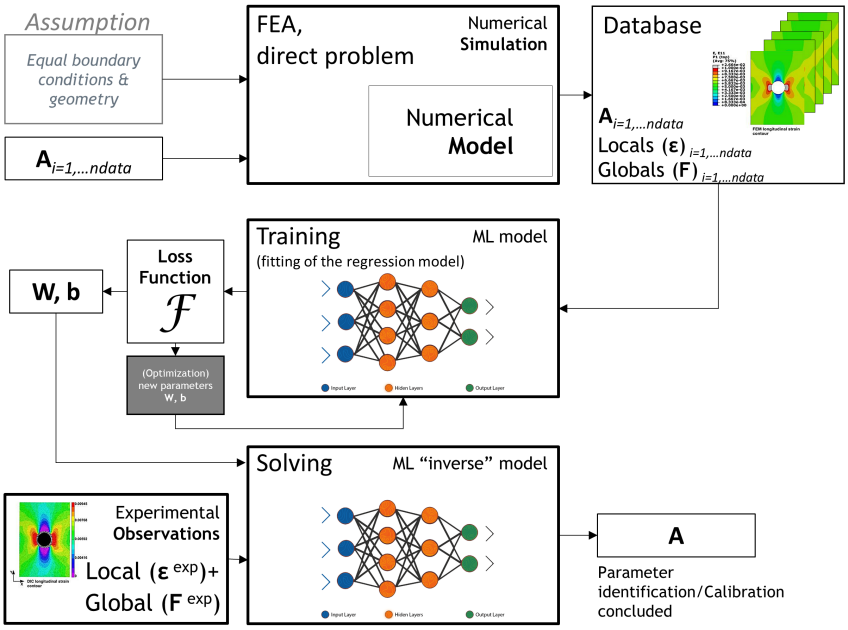

Figure 4. Schematic flowchart for the ML method.

ML models are fitted (the training phase) with gradient-based algorithms because their topology is well known and their derivatives can be easily calculated by Automatic Differentiation (AD). ADAM, a stochastic gradient descent algorithm, is one of the most used optimization techniques [16]. Nevertheless, least-square algorithms are still being used.

5. Comparison of the parameter identification in non-linear elastoplasticity

In order to compare the different presented approaches, a simple mechanical heterogeneous specimen is used for the identification of the hardening of a isotropic constitutive model. It is assumed that the elastoplastic material model (i) has the elastic behaviour described by the linear Hook law, where Young's modulus and Poisson's ratio are defined by $E = 210$ GPa and $\nu = 0.3$, and (ii) the plastic yield is governed by the von Mises yield criterion and hardening is described by the Swift's law, given by

$$\sigma_{\mathrm{Y}} = k\left(\varepsilon_0 + \bar{\varepsilon}^{\mathrm{p}}\right)^n, \quad \text{with} \quad \varepsilon_0 = \left(\frac{\sigma_0}{k}\right)^{1/n}, \tag{8}$$

where σ_{Y} is the flow stress, σ_0 is the initial yield stress, k and n are the hardening coefficient and exponent, respectively, and $\bar{\varepsilon}^{\mathrm{p}}$ is the equivalent plastic strain. The three material parameters to be identified are $\mathbf{A} = \{k, n, \sigma_0\}$.

5.1. Heterogeneous dog-bone test

For this study, synthetic data from a virtual experiment is used for all three approaches. This procedure has the advantage of knowing *a priori* the solution, as ground truth, although some would suggest not to use this methodology [31].

A heterogeneous specimen with smooth cross-section change is selected. This test, here called heterogeneous dog bone, was retrieved from [22, 15] and can be seen in Figure 5. For the problem, only one quarter is used due to symmetries conditions in $x = 0$ and $y = 0$. The model, numerically created using Abaqus standard, assumes the plane stress formulation and uses a regular mesh of 112 CPS4 (bilinear and full integration) elements. A vertical displacement of

3 mm is applied on the top of the specimen and 30 time increments were used. The virtual material is characterised by the parameter set $\mathbf{A} = \{565, 0.26, 160\}$. An equivalent plastic strain of 0.25 can be observed in the specimen's centre and the deformation gradient is observed mainly in the vertical direction. Although this test is heterogeneous, only hardening heterogeneity is observed. No anisotropy is accounted and all specimen yields in the same $\sigma_1 - \sigma_2$ yield locus direction.

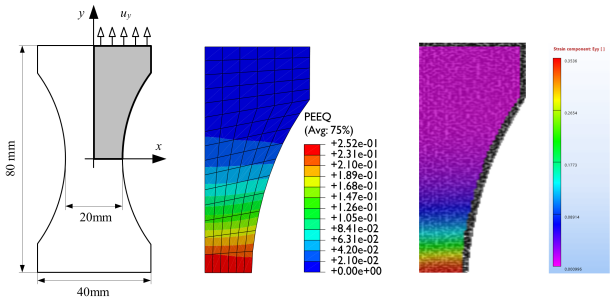

Figure 5. Specimen geometry and corresponding finite element model and DIC measurement. Equivalent Plastic strain distribution after a displacement of 3 mm. The test was retrieved from [22].

5.2. Results and discussion

Implementation details for the three inverse problem approaches can be seen in [22, 23] and [2]. For all approaches, the logarithmic strain was retrieved from the centroid of the element, and the identical coordinates for the strain points are considered for the experimental and the numerical values, avoiding interpolation errors. The hardening model is implemented in both FEA (for the FEMU and the training in ML) and the VFM by a user subroutine. For the VFM, the simplest virtual fields were chosen: $u*_x = 0$ and $u^*_y = 0$. These only two VFs were sufficient for the correct determination of the parameter. Nevertheless, multiple VFs or special determined VFs (e.g. using the sensitive-based VFs) would result in even better results, with the cost of larger CPU time.

For the ML approach, a gradient boosted trees algorithm, called XGBoost and proposed by Chen and Guestrin [7], is used. It can be shown that this model is more efficient than the ANN, which is the most used in ML [30]. However, it still needs a training data set for the three material parameters. Here, the database was limited to 5000 combinations of the parameters, which were retrieved using the Latin Hypercube Sampling (LHS) [3], and using the following universe: $\mathbf{A} \in \{[280, 700], [0.1, 0.3], [80, 300]\}$.

Considering that the Levenberg-Marquardt optimization algorithm is starting set dependent, two initial parameters sets are used for both the FEMU and the VFM approaches. These can be seen in Table 1 and are denoted as superior and inferior due to their values. However, from the obtained results, it can be seen that the VFM method is not as sensitive to the initial parameters set as the FEMU approach. Both starting sets have led the VFM to the same final parameters, whose errors are very low. These are not zero because the virtual experiment was retrieved using FEA and not using the VFM itself. But the fact that both starting sets lead to the exact same results shows that it is the global minimum for the VFM.

The FEMU has also reached good results for both initial sets, but their difference shows the method's sensitivity to the initial dependence and more prone to local minima. The magnitude of the parameter difference to the virtual material parameters of the FEMU is similar to the VFM, however, it took more than ten times to reach the final results, confirming the large CPU efficiency of the VFM method.

Table 1. Initial sets and identified parameters for the three approaches (FEMU, VFM and ML) presented.

$\mathbf{A} =$	k [MPa]	n	σ_0 [MPa]	Obj. function $\mathcal{L}(\mathbf{A})$
$\mathbf{A}^{\text{sup}}$	965	0.35	234	—
$\mathbf{A}^{\text{inf}}$	165	0.08	100	—
$\mathbf{A}^{\text{VFM}}$	564.24	0.265	160.72	2.29×10^{-5}
Error [%]	0.134	1.923	0.450	—
$\mathbf{A}^{\text{FEMU, sup}}$	568.74	0.2633	160.36	5.67×10^{-6}
Error [%]	0.662	1.269	0.225	—
$\mathbf{A}^{\text{FEMU, inf}}$	552.77	0.250	159.7	7.95×10^{-5}
Error [%]	2.164	3.729	0.187	—
$\mathbf{A}^{\text{ML}}$	564.67	0.259	160.12	1.46×10^{-5}
Error [%]	0.058	0.385	0.075	—

The ML approach seems to be effective, and the errors were not larger than the ones from the other competitor approaches. It has the advantage of total non-dependence of the starting set because it does not require an optimization procedure for the identification. If the time for training is not considered, the ML approach is, without doubt, the most efficient because the solution is obtained instantaneously. However, training requires a quite large database, here with 5000 direct problem solutions, which took around 7 days to be fulfilled. The training itself took only 50 minutes for 5000 samples and 6020 features.

Nevertheless, it should be noticed that the influence of the size of the database was analysed, using database dimensions of 750, 1000, 1500, 2500, and 4500, and it was verified that a database of only 1500 samples could deliver MSE smaller than 2% in training. This result is even more enhanced if PCA analysis is introduced and only 450 samples could be used without loss of quality. For more details, see [2].

Figure 6 shows that the hardening with the parameters obtained by the FEMU, VFM and ML approaches is similar, however, some small differences can be seen for the hardening identified by the ML approach.

6. Concluding remarks

In this paper, three approaches for parameter identification in non-linear elastoplasticity were compared and discussed. Although the FEMU and the VFM are regularly used in the scientific communities, ML approaches are not so usual. The results here presented are from the identification of a 3-parameter non-linear hardening model. Even though the case study is not very demanding, the advantages and disadvantages of each approach could be disclosed. The advantage of the ML is not being starting-set and optimization dependent but requires large computational effort for the database building and training. The VFM is clearly very computational effective and robust, yet it requires a wiser implementation. The FEMU is, without doubt, the simplest and wide-range method to work with. But its optimization dependence and effort are major drawbacks. Therefore, the three methods present disadvantages that maybe could be eliminated if a hybrid method or redundant strategy would be used.

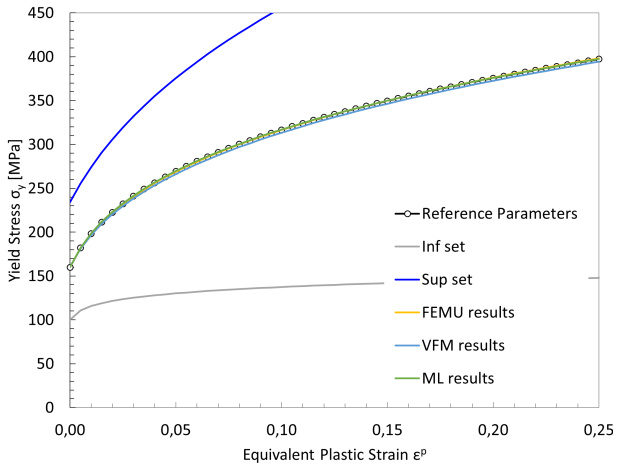

Figure 6. Hardening curve for the parameters obtained with the FEMU, VFM and ML approaches. Comparison with the virtual material Swift hardening law.

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